Solving the Stable Marriage Problem using Communicating Answer Set Programming

Sofie De Clercq[a]

Steven Schockaert[b], Martine De Cock[a], Ann Nowé[c]

[a] Dept. of Appl. Mathematics and Computer Science, Ghent University, Belgium
[b] School of Computer Science and Informatics, Cardiff University, UK
[c] Computational Modeling Lab, Vrije Universiteit Brussel, Belgium

UNIVERSITEIT GENT

FWO VLAANDEREN
Fonds Wetenschappelijk Onderzoek
Research Foundation – Flanders

Content of the talk

1. The Stable Marriage Problem (SMP)

Classical SMP

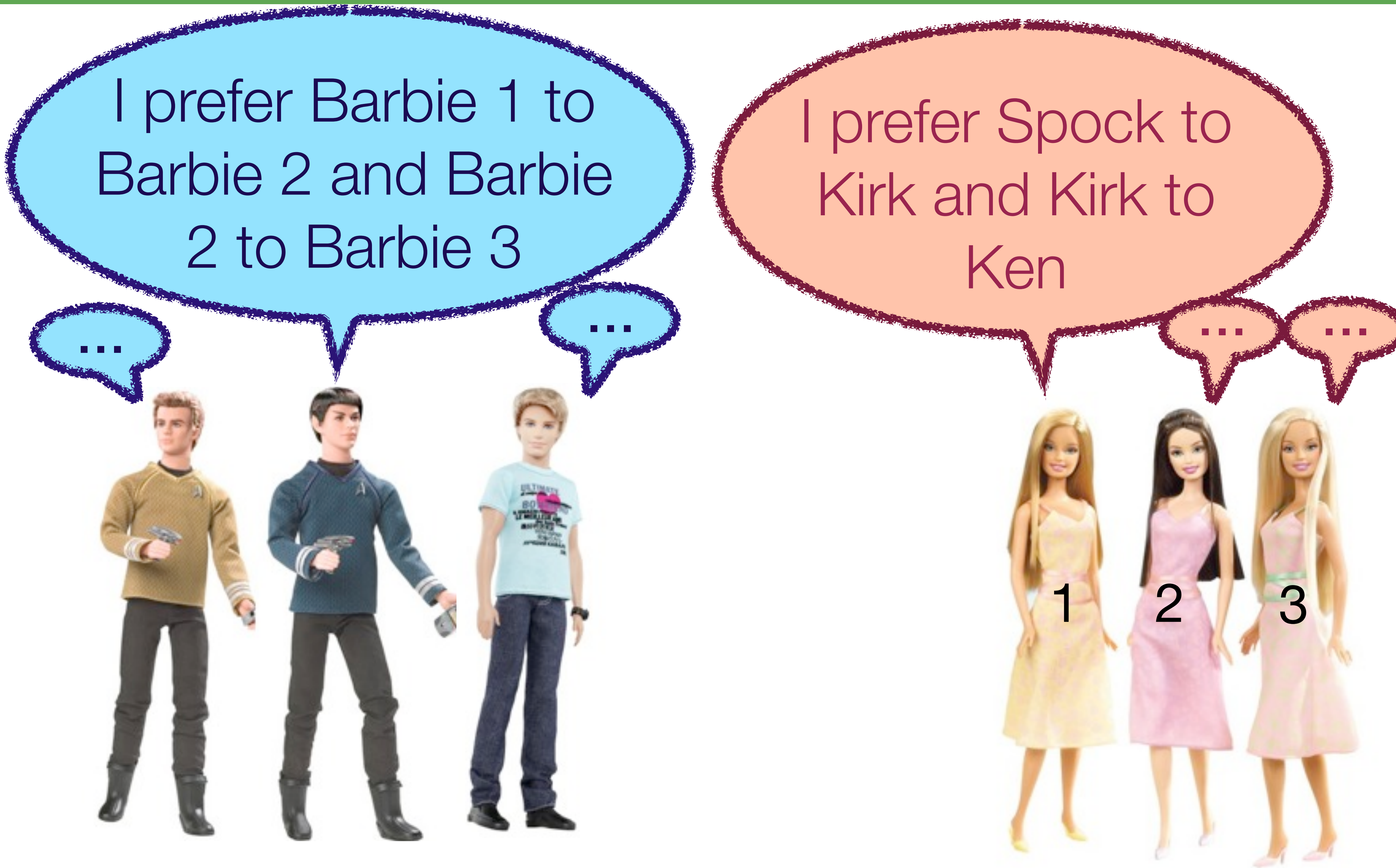
SMP with unacceptability

SMP with unacceptability and indifference

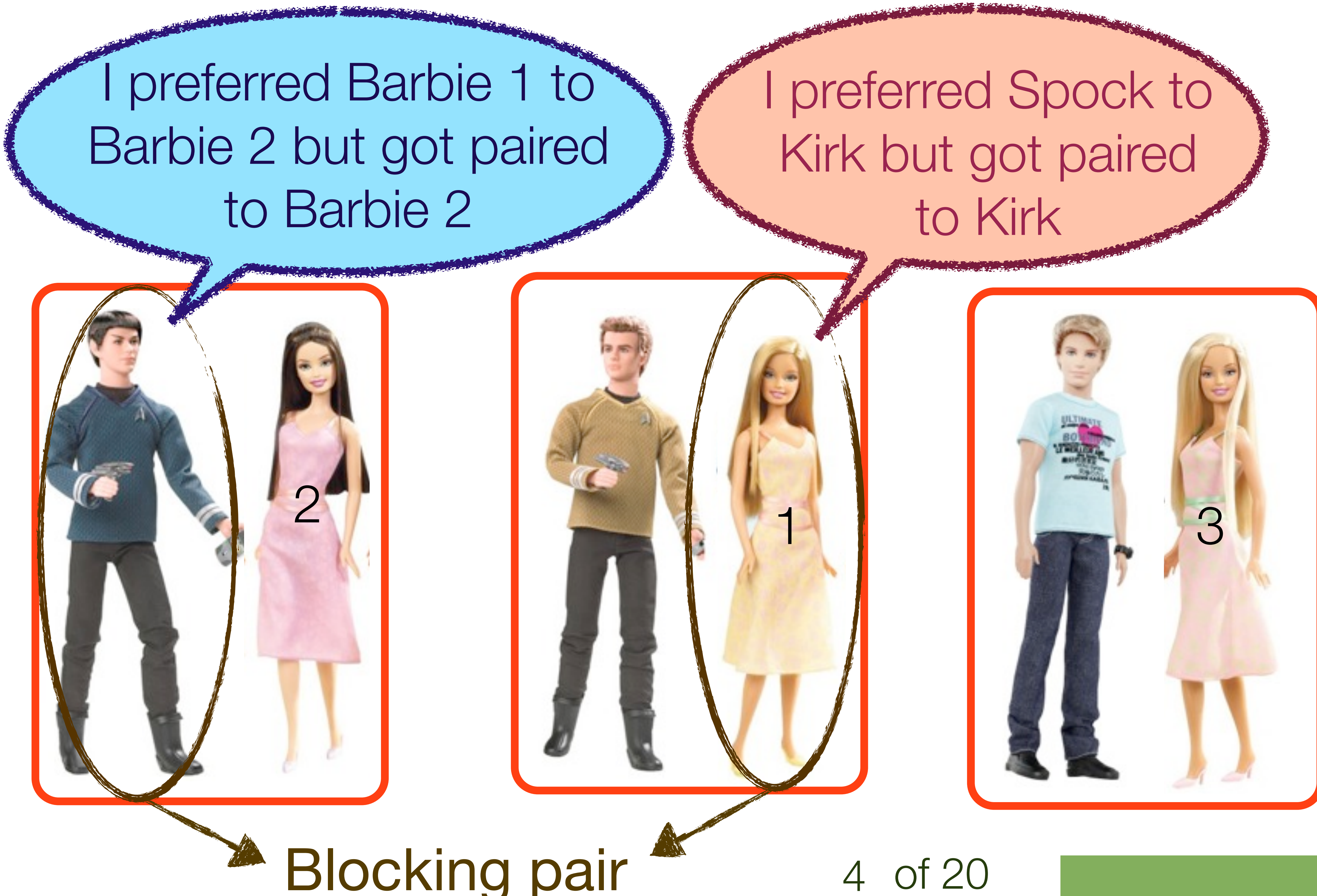
2. Communicating Answer Set Programming (CASP)

3. Modelling and solving the SMP in CASP

The Stable Marriage Problem



The Stable Marriage Problem



The Stable Marriage Problem

Goal of the SMP: find a stable set of marriages

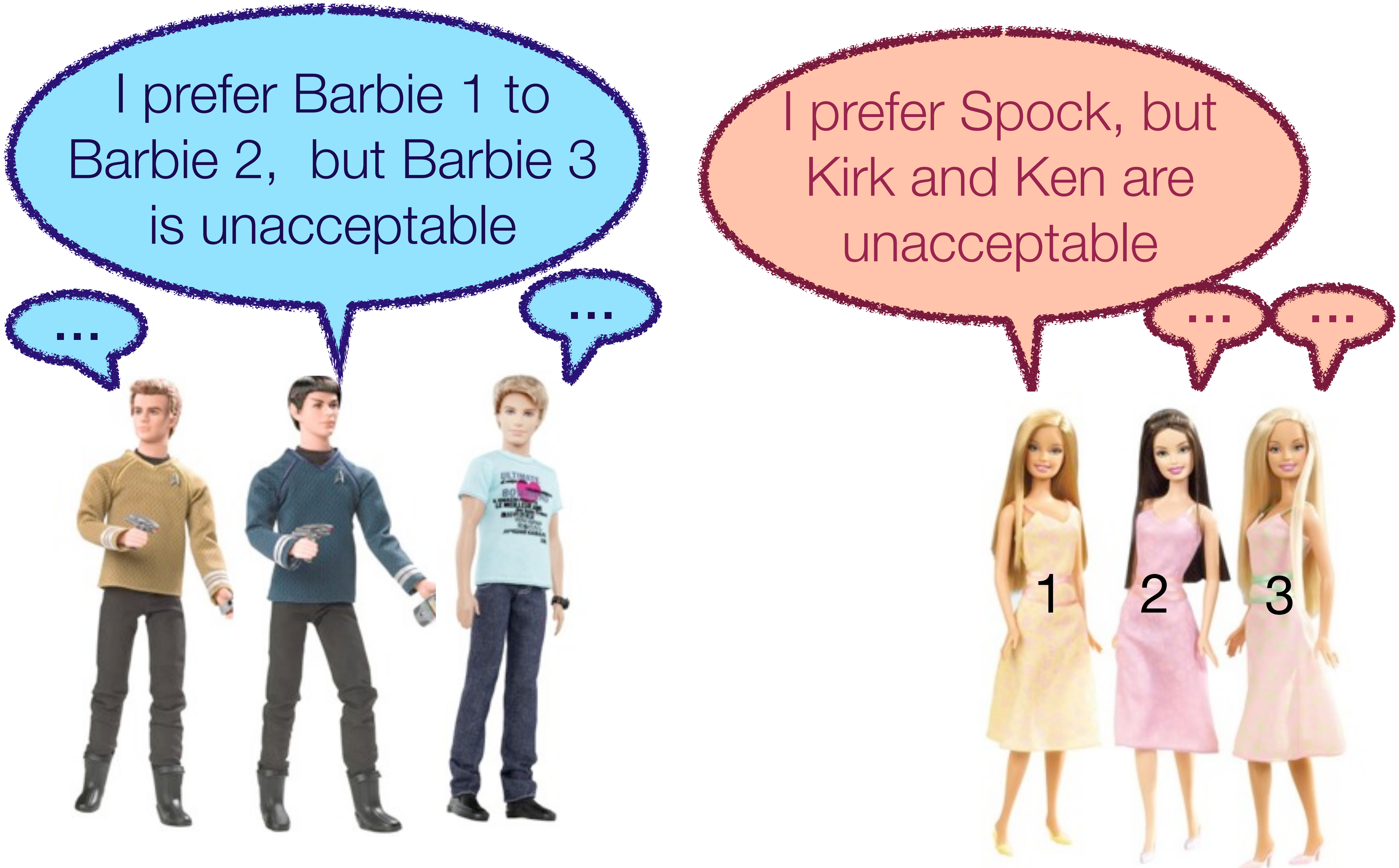
without blocking pairs

always exists

Classical SMP: as many men as women, strict and complete preferences

SMP with unacceptability: strict but incomplete preferences allowing the men and women to point out unacceptable partners

SMP with unacceptability



SMP with unacceptability

Blocking individual

I found Kirk unacceptable but got paired to Kirk

The Stable Marriage Problem

Goal of the SMP: find a stable set of marriages

without blocking pairs

always exists

Classical SMP: as many men as women, strict and complete preferences

---

SMP with unacceptability: strict but incomplete preferences allowing the men and women to point out inacceptable partners

---

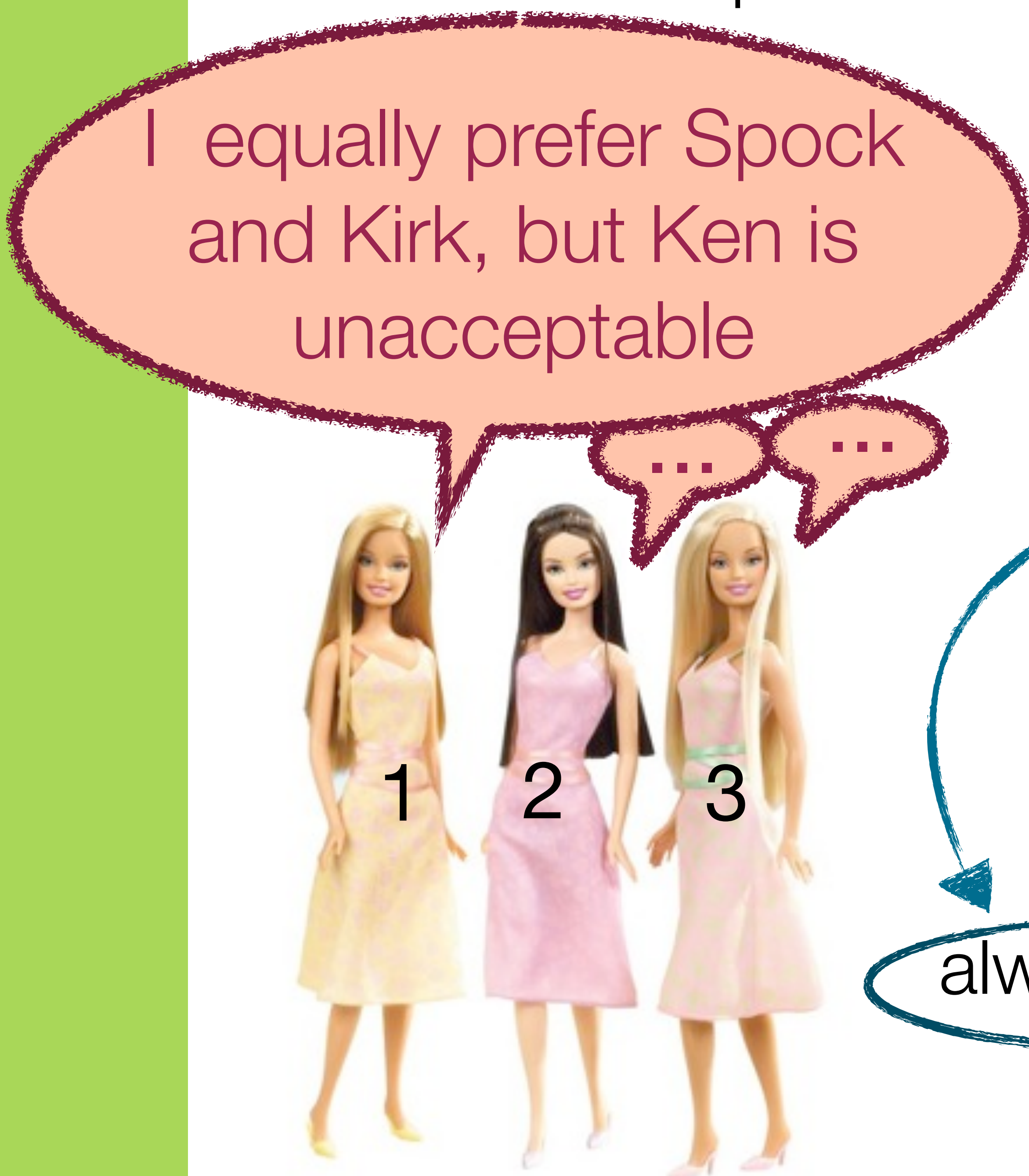
Goal of the SMP with unacceptability: find a stable set of marriages

without blocking pairs and blocking individuals

always exists

SMP with unacceptability and indifference

SMP with unacceptability and indifference: incomplete preferences, not necessarily strict



Goal of the SMP with unacceptability and indifference: find a stable set of marriages

without blocking individuals and blocking pairs i.e. a man and woman who strictly prefer each other to their actual partner

always exists

Content of the talk

1. The Stable Marriage Problem (SMP)

Classical SMP

SMP with unacceptability

SMP with unacceptability and indifference

2. Communicating Answer Set Programming (CASP)

3. Modelling and solving the SMP in CASP

(Communicating) ASP

CASP = Communicating Answer Set Programming

ASP program: finite set of rules of the form

$$\underbrace{a}_{\text{head}} \leftarrow \underbrace{b_1, \ldots, b_n}_{\text{positive body}}, \underbrace{not\, c_1, \ldots, not\, c_m}_{\text{negative body}}$$

negation-as-failure: $not\, c_m$ is true if no proof for c_m

Goal of ASP: program rules describe the problem and answer sets provide the solutions of the problem

Answer sets of ASP programs

Example: answer sets of an ASP program

$a \leftarrow not\, b$

$b \leftarrow not\, a$

guess		derived from ASP program		
$\{\}$	$\neq$	$\{a, b\}$	→	no answer set
$\{a\}$	$=$	$\{a\}$	→	answer set
$\{b\}$	$=$	$\{b\}$	→	answer set
$\{a, b\}$	$\neq$	$\{\}$	→	no answer set

Communicating ASP

CASP program: unit of communicating ASP programs (components), i.e. finite set of rules of the following form:

$$A{:}a \leftarrow B_1{:}b_1, \ldots, B_n{:}b_n, not\, C_1{:}c_1, \ldots, not\, C_m{:}c_m$$

There is proof that a is true in component A

There is no proof that c_m is true in component C_m

This rule belongs to component A

Answer sets of CASP programs

Example: answer sets of a CASP program $\mathcal{P}$

$$Q{:}a \leftarrow R{:}a, not\, Q{:}b \qquad R{:}b \leftarrow not\, Q{:}a$$
$$Q{:}b \leftarrow Q{:}a, R{:}b \qquad R{:}a \leftarrow Q{:}b$$

$I = \{Q{:}b, R{:}a\}$ answer set of $\mathcal{P}$?

Answer sets of CASP programs

Example: answer sets of a CASP program $\mathcal{P}$

$$Q{:}a \leftarrow R{:}a, not\, Q{:}b \qquad R{:}b \leftarrow not\, Q{:}a$$
$$Q{:}b \leftarrow Q{:}a, R{:}b \qquad R{:}a \leftarrow Q{:}b$$

$I = \{Q{:}b, R{:}a\}$ answer set of $\mathcal{P}$?

$\Leftrightarrow$ $I_{\downarrow Q} = \{b\}$ answer set of Q^I and $I_{\downarrow R} = \{a\}$ answer set of R^I

reduct ↙ reduct ↙

Answer sets of CASP programs

Example: answer sets of a CASP program $\mathcal{P}$

$Q{:}a \leftarrow R{:}a, not\, Q{:}b$ $\quad$ $R{:}b \leftarrow not\, Q{:}a$

$Q{:}b \leftarrow Q{:}a, R{:}b$ $\quad$ $R{:}a \leftarrow Q{:}b$

$I = \{Q{:}b, R{:}a\}$ answer set of $\mathcal{P}$?

$\Leftrightarrow$ $I_{\downarrow Q} = \{b\}$ answer set of Q^I and $I_{\downarrow R} = \{a\}$ answer set of R^I

reduct

delete this rule because $Q{:}b$ is in I

delete this rule because $R{:}b$ is not in I

reduct

remove this part because $Q{:}a$ is not in I

remove this part because $Q{:}b$ is in I

Answer sets of CASP programs

Example: answer sets of a CASP program $\mathcal{P}$

$$Q{:}a \leftarrow R{:}a, not\, Q{:}b \qquad R{:}b \leftarrow not\, Q{:}a$$
$$Q{:}b \leftarrow Q{:}a, R{:}b \qquad R{:}a \leftarrow Q{:}b$$

$I = \{Q{:}b, R{:}a\}$ answer set of $\mathcal{P}$?

$\Leftrightarrow$ $I_{\downarrow Q} = \{b\}$ answer set of Q^I and $I_{\downarrow R} = \{a\}$ answer set of R^I

reduct

delete this rule because $Q{:}b$ is in I

delete this rule because $R{:}b$ is not in I

$\Rightarrow$ Q^I contains no rules

$\Rightarrow$ $I_{\downarrow Q}$ is no answer set of Q^I

$\Rightarrow$ I is no answer set of $\mathcal{P}$

reduct

remove this part because $Q{:}a$ is not in I

remove this part because $Q{:}b$ is in I

$\Rightarrow$ $R^I = \{b \leftarrow, a \leftarrow\}$

$\Rightarrow$ $I_{\downarrow R}$ is no answer set of R^I

Answer sets of CASP programs

Example: answer sets of a CASP program $\mathcal{P}$

$$Q{:}a \leftarrow R{:}a, not\, Q{:}b \qquad R{:}b \leftarrow not\, Q{:}a$$

$$Q{:}b \leftarrow Q{:}a, R{:}b \qquad R{:}a \leftarrow Q{:}b$$

$I = \{R{:}b\}$ answer set of $\mathcal{P}$?

Answer sets of CASP programs

Example: answer sets of a CASP program $\mathcal{P}$

$$Q{:}a \leftarrow R{:}a, not\, Q{:}b \qquad R{:}b \leftarrow not\, Q{:}a$$

$$Q{:}b \leftarrow Q{:}a, R{:}b \qquad R{:}a \leftarrow Q{:}b$$

$I = \{R{:}b\}$ answer set of $\mathcal{P}$?

$\Leftrightarrow$ $I_{\downarrow Q} = \{\}$ answer set of Q^I and $I_{\downarrow R} = \{b\}$ answer set of R^I

reduct ↗ reduct ↗

Answer sets of CASP programs

Example: answer sets of a CASP program $\mathcal{P}$

$Q{:}a \leftarrow R{:}a, not\, Q{:}b$ $\qquad$ $R{:}b \leftarrow not\, Q{:}a$

$Q{:}b \leftarrow Q{:}a, R{:}b$ $\qquad$ $R{:}a \leftarrow Q{:}b$

$I = \{R{:}b\}$ answer set of $\mathcal{P}$?

$\Leftrightarrow \quad I_{\downarrow Q} = \{\}$ answer set of Q^I and $I_{\downarrow R} = \{b\}$ answer set of R^I

reduct

delete this rule because $R{:}a$ is not in I

remove this part because $R{:}b$ is in I

reduct

remove this part because $Q{:}a$ is not in I

delete this rule because $Q{:}b$ is not in I

Answer sets of CASP programs

Example: answer sets of a CASP program $\mathcal{P}$

$$Q{:}a \leftarrow R{:}a, not\, Q{:}b \qquad R{:}b \leftarrow not\, Q{:}a$$
$$Q{:}b \leftarrow Q{:}a, R{:}b \qquad R{:}a \leftarrow Q{:}b$$

$I = \{R{:}b\}$ answer set of $\mathcal{P}$?

$\Leftrightarrow \quad I_{\downarrow Q} = \{\}$ answer set of Q^I and $I_{\downarrow R} = \{b\}$ answer set of R^I

reduct

delete this rule because $R{:}a$ is not in I

remove this part because $R{:}b$ is in I

$\Rightarrow \quad Q^I = \{b \leftarrow a\}$

$\Rightarrow \quad I_{\downarrow Q}$ is an answer set of Q^I

$\Rightarrow \quad I$ is an answer set of $\mathcal{P}$

reduct

remove this part because $Q{:}a$ is not in I

delete this rule because $Q{:}b$ is not in I

$\Rightarrow R^I = \{b \leftarrow\}$

$\Rightarrow \quad I_{\downarrow R}$ is an answer set of R^I

Content of the talk

1. The Stable Marriage Problem (SMP)

Classical SMP

SMP with unacceptability

SMP with unacceptability and indifference

2. Communicating Answer Set Programming (CASP)

3. Modelling and solving the SMP in CASP

SMP in CASP

Gale-Shapley algorithm to calculate a stable set of marriages

Using CASP to model and solve the SMP and its variants

- transparent model
- easily extending to other variants
- easily adding constraints

Three components for CASP Program $\mathcal{P}$:

- M: to model the intentions of the men,
- W: to model the intentions of the women,
- A: to model the acceptance of each other as partners.

SMP in CASP

$$M{:}propose(sp, b_1) \leftarrow$$

$$M{:}propose(sp, b_2) \leftarrow not\ A{:}accept(sp, b_1)$$

$$M{:}propose(sp, b_3) \leftarrow not\ A{:}accept(sp, b_1), not\ A{:}accept(sp, b_2)$$

$$M{:}accept(sp, sp) \leftarrow not\ A{:}accept(sp, b_1), not\ A{:}accept(sp, b_2), not\ A{:}accept(sp, b_3)$$

I prefer Barbie 1 to Barbie 2 and Barbie 2 to Barbie 3

$$A{:}accept(sp, b_1) \leftarrow M{:}propose(sp, b_1), W{:}propose(sp, b_1)$$

I equally prefer Spock and Kirk, but Ken is unacceptable

1

$$W{:}propose(sp, b_1) \leftarrow not\ A{:}accept(ki, b_1)$$

$$W{:}propose(ki, b_1) \leftarrow not\ A{:}accept(sp, b_1)$$

$$W{:}accept(b_1, b_1) \leftarrow not\ A{:}accept(sp, b_1), not\ A{:}accept(ki, b_1)$$

Answer sets and stable sets

Connection between the answer set of a CASP program induced by an SMP instance and the stable sets of marriages of that SMP instance:

- For every stable set $I = \{(m_1, w_1), (m_2, w_2), \ldots\}$ of marriages of a certain SMP instance, the CASP program induced by it has an answer set which contains $A{:}accept(m, w)$ for every (m, w) in I.
- Conversely, for every answer set I of the CASP program induced by a certain SMP instance, the set of marriages consisting of the couples (m, w) for which $A{:}accept(m, w)$ is in I is a stable set of marriages.

→ The CASP program induced by an SMP instance solves the SMP instance

Future work

- Using CASP to find ‘optimal’ stable matches (matches that are more preferred than others)
- Extending CASP with utilities to fit cooperative games

Any questions?

contact: SofieR.DeClercq@UGent.be