Towards Possibilistic Fuzzy Answer Set Programming

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Possibilistic Fuzzy Answer Set Programming

motivation

Combine

- non-monotonicity,
- continuous domains, and
- uncertainty

in a single framework.

example

It is more certain that we will have a barbecue when there is more sunshine and the size of the barbecue is determined by how hungry we are.
Answer Set Programming (ASP)
simple programs

Simple ASP program is set of simple rules of the form

\[ \overbrace{l_0}^{\text{head}} \leftarrow \overbrace{l_1, \ldots, l_m}^{\text{body}} \]

on which we apply forward chaining.

example

\[ \text{rushHour} \leftarrow \]
\[ \text{raining} \leftarrow \]
\[ \text{longExpectedDrivingTime} \leftarrow \text{rushHour, raining} \]
Fuzzy Answer Set Programming (FASP)

overview

Generalization of ASP to continuous domains

literals associated with a truth degree from unit interval $[0, 1]$; e.g. sunny$^{0.6}$

example

\[\begin{align*}
\text{rushHour} & \leftarrow 1 \\
\text{raining} & \leftarrow 0.2 \\
\text{longExpectedDrivingTime} & \leftarrow f(\text{rushHour}, \text{raining})
\end{align*}\]

$f$ allows to encode exact relationship
Fuzzy Answer Set Programming (FASP) shortcomings

Only a generalization of ASP to continuous domains, not a form of approximate reasoning

need uncertainty
- well studied in ASP (possibility, probability ...)
- studied for discrete domains

why combine FASP with possibility theory?
- strong link fuzziness and possibility
- combination allows approximate reasoning
- computationally interesting
Possibilistic Fuzzy ASP (PFASP)

overview

rules of the form

\[ c : l_0 \leftarrow f(l_1, \ldots, l_m) \]

ability to represent vague/fuzzy information:

example

\[
\begin{align*}
0.8 & : \text{tumor} \leftarrow 0.1 & 0.8 & : \text{tumor} \leftarrow 0.4 \\
0.6 & : \text{tumor} \leftarrow 0.2 & 0.6 & : \text{tumor} \leftarrow 0.6 \\
0.4 & : \text{tumor} \leftarrow 0.3 & 0.4 & : \text{tumor} \leftarrow 0.8 \\
\text{“small tumor”} & & \text{“mid-sized tumor”}
\end{align*}
\]
A finite set of atoms;
literal \( l \) either \( a \in A \) or \( \neg a \);
\( \mathcal{L} \) set of all literals;
interpretation \( L \) is consistent subset of \( \mathcal{L} \)

immediate consequence operator for simple rules:
\[
T_P(L) = L \cup \{l_0 \mid (l_0 \leftarrow l_1, \ldots, l_m) \in P \land \{l_1, \ldots, l_m\} \subseteq L\}
\]
answer set iff \( L = \) least fixpoint \( P^\ast \) starting from \( \emptyset \)
Deriving Conclusions in ASP
fixpoint theory – example

example

\[ \text{rushHour} \leftarrow \]
\[ \text{raining} \leftarrow \]
\[ \text{longTime} \leftarrow \text{rushHour}, \text{raining} \]

\[
T_P(\emptyset) = \emptyset \cup \{\text{rushHour, raining}\}
\]
\[
T_P(\{\text{rushHour, raining}\}) = \{\text{rushHour, raining}\} \cup \{\text{longTime}\}
\]

\[ P^* = \{\text{rushHour, raining, longTime}\} \]
Deriving Conclusions in FASP

fixpoint theory

fuzzy interpretation is consistent mapping $L : \mathcal{L} \rightarrow [0, 1]$

$[f(l_1, ..., l_m)]_L = f(L(l_1), ..., L(l_m))$

fuzzy interpretation $L$ is model of

- rule $r = l_0 \leftarrow l_1, ..., l_m$, denoted as $L \models r$,
  if $L(l_0) \geq [f(l_1, ..., l_m)]_L$

- program $P$ if $L \models r$ for all $r \in P$

immediate consequence operator:

$T_P(L)(l_0) = \sup \{ [f(l_1, ..., l_m)]_L \mid l_0 \leftarrow f(l_1, ..., l_m) \in P \}$

answer set iff $L$ is minimal model of $P$
Deriving Conclusions in FASP

fixpoint theory – example

\begin{verbatim}
example

slippery ← 0.1
raining ← 0.4
slippery ← raining

\end{verbatim}

\begin{verbatim}
example

TP(\{slippery^0, raining^0\})(slippery) = sup\{[0.1]_L\}
TP(\{slippery^{0.1}, raining^{0.4}\})(slippery) = sup\{[0.1]_L, [raining]_L\}
= sup\{0.1, 0.4\}

\end{verbatim}

answer set is \{slippery^{0.4}, raining^{0.4}\}
Possibilistic Fuzzy ASP (PFASP)

PFASP language

\( V \) is a valuation \( V : \mathcal{L} \rightarrow (C \rightarrow [0, 1]) \)

intuitive meaning
\[ V(l)(c) = d \]
derive with certainty \( c \) that the truth degree of literal \( l \) is at least \( d \)

\( V(l)(c) = d \) also denoted as \( l^{c,d} \)
Possibilistic Fuzzy ASP (PFASP)
definitions

**some definitions**
- $V^c$ is fuzzy set $V^c(l) = V(l)(c)$
- $V$ is interpretation iff $\forall c \in C \cdot V^c$ consistent

possibilistic fuzzy simple rule is a pair $(n(r), r)$ with $r$ a fuzzy simple rule and $n(r)$ certainty value associated with $r$, denoted as

$$n(r) : l_0 \leftarrow f(l_1, ..., l_m)$$

$$P_c = \{(r, n(r)) \in P \mid n(r) \geq c\}$$
Possibilistic Fuzzy ASP (PFASP)

fixpoint definition

\[ T_P(L)(l)(c) = \sup\{ [\alpha]_{L^c} \mid (r : l \leftarrow \alpha) \in P_c \} \]

monotonic operator, thus has least fixpoint \( P^* \)

interpretation \( L \) is answer set of \( P \) iff \( L = P^* \)
Possibilistic Fuzzy ASP (PFASP)

example

1: cold ← 0.6
1: wet ← 0.4
1: risky ← cold \cdot snow
0.8: snow ← (cold \geq 0.5) \land wet
0.6: risky ← 0.5 \cdot cold
0.6: risky ← 0.8 \cdot wet
Possibilistic Fuzzy ASP (PFASP)

example – first step

example

1: \textit{cold} \leftarrow 0.6
1: \textit{wet} \leftarrow 0.4
1: \textit{risky} \leftarrow \textit{cold} \cdot \textit{snow}
0.8: \textit{snow} \leftarrow (\textit{cold} \geq 0.5) \land \textit{wet}
0.6: \textit{risky} \leftarrow 0.5 \cdot \textit{cold}
0.6: \textit{risky} \leftarrow 0.8 \cdot \textit{wet}

\[ T_P(\emptyset) = S_0 = \{ \textit{cold}^{1;0.6}, \textit{wet}^{1;0.4} \} \]
Possibilistic Fuzzy ASP (PFASP)
example – second step

\[ T_P(S_0) = \{ \textit{cold}^{1.0.6}, \textit{wet}^{1.0.4}, \textit{snow}^{0.8.0.4}, \textit{risky}^{0.6.0.32} \} \]
Possibilistic Fuzzy ASP (PFASP)

example – last step

example

1: \textit{cold} \leftarrow 0.6
1: \textit{wet} \leftarrow 0.4
1: \textit{risky} \leftarrow \textit{cold} \cdot \textit{snow}
0.8: \textit{snow} \leftarrow (\textit{cold} \geq 0.5) \land \textit{wet}
0.6: \textit{risky} \leftarrow 0.5 \cdot \textit{cold}
0.6: \textit{risky} \leftarrow 0.8 \cdot \textit{wet}

\[ TP(S_1) = P^* = \{ \textit{cold}^{1;0.6}, \textit{wet}^{1;0.4}, \textit{snow}^{0.8;0.4}, \]
\[ \textit{risky}^{0.6,0.32}, \textit{risky}^{0.8,0.24} \} \]
Possibilistic Fuzzy ASP (PFASP)
model theory

so far syntactic fixpoint
does not readily lead to actual implementation

model
interpretation \( L \) is a model of PFASP simple
rule of the form \( r = c : l \leftarrow body \), denoted
\( L \models r \), iff \( \forall c \in C \cdot M(l)(c) \geq [body]_{L_c} \);
model of PFASP program \( P \) iff \( \forall r \in P : L \models r \)

Proposition
minimal model is \( M \) defined as
\( \forall l \in \mathcal{L} \cdot (\forall c \in C) \cdot M(l)(c) = F_c(l) \)
with \( F_c \) minimal model of fuzzy program \( P_c \)
Syntactic Extensions
approximate conditions

extensions to write programs more succinctly/naturally

approximation rule

\[
l \leftarrow \text{approx}^f(\text{body})
\]

shorthand for

\[
\{ c : l \leftarrow \text{body} \otimes f(c) \mid c \in C \}
\]

deal with approximate information, i.e. no full confidence in information: either weaken the certainty or weaken the body
Syntactic Extensions
approximate conditions – example

example
We tend to be happy when it is warm/sunny.

\[
\text{happy} \leftarrow \text{approx}^f (\text{warm} \otimes \text{sunny}) \quad (f(x) = 1.2 - x)
\]

1 : warm \leftarrow 0.8
1 : sunny \leftarrow 0.6

conclusion: ..., happy^{0.48;0.2}, ..., happy^{0.096;1.0}

allows for approximate reasoning (see paper)
Syntactic Extensions
variable certainty weights

uncertainty qualifying rules

\[ f_1(l_1, \ldots, l_n) : \text{let } l \leftarrow f_2(l'_1, \ldots, l'_m) \]

shorthand for

\[ \{ c : \text{let } l \leftarrow f_2(l'_1, \ldots, l'_m) \wedge ((f_1(l_1, \ldots, l_n) \geq c) \mid c \in C) \} \]

certainty is not a constant, but based on degree to which (part of) the body is true
Syntactic Extensions
variable certainty weights – example

**example**
It is more certain that we will have a barbecue when there is more sunshine and the size of the barbecue is determined by how hungry we are.

sunshine : $bbq \leftarrow hungry$
1 : $sunshine \leftarrow 0.9$
1 : $hungry \leftarrow 0.2$

unique answer set:
$\{hungry^{0.2}, sunshine^{0.9}, bbq^{0.9,0.2}\}$
Conclusion

what we did:

- possibilistic fuzzy ASP rules/literals as extension of fuzzy rules/literals with necessity degree
- syntactic and semantic approach
- readily implementable on fuzzy ASP solvers
- interesting syntactic extensions

where is non-monotonicity (NAF)?
current semantics for NAF in possibilistic ASP are problematic/counter-intuitive in general setting

Thank you for your attention!