# Communicating ASP and the Polynomial Hierarchy

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May 18, 2011

# Communicating ASP and the Polynomial Hierarchy

overview

overview of this talk:

- background
  what are communicating programs?
- multi-focused answer sets
  means for local minimality
- semantics
- expressiveness

# Communicating ASP and the Polynomial Hierarchy

background

communicating ASP defined as:

- many individual ASP programs …
  - ↳ name each *component program*
    e.g. $Q, R, \ldots$
- … that can communicate with each other
  - ↳ *situated literals*
    e.g. $Q:a, \ldots$
    ||

"ask to $Q$ whether it believes that the atom $a$ is true"

# Communicating ASP and the Polynomial Hierarchy

## background

communicating ASP is:

- … not a new idea
  used by Roelofsen, Brewka, Eiter …
  also known as bridge rules

- … open for interpretation?

$$Q : b \leftarrow Q : a$$
$$Q : a \leftarrow Q : b$$

→ $\{\}$

→ ~~$\{Q : a, Q : b\}$~~

equivalent to

$$a \leftarrow b$$
$$b \leftarrow a$$

communicating ASP is a generalization of classical ASP

# Communicating ASP and the Polynomial Hierarchy

## background

communicating ASP is:

- … not a new idea
  used by Roelofsen, Brewka, Eiter …
  also known as bridge rules

- … open for interpretation!

$$Q\!:\!a \leftarrow R\!:\!a$$
$$R\!:\!a \leftarrow Q\!:\!a$$

$\{\}$

$\{Q\!:\!a, R\!:\!a\}$

this circularity identified
as being essential to
represent social reasoning
e.g. Buccafurri2008

- … more expressive ← induces a guess
  ↳ NP-complete for definite component programs

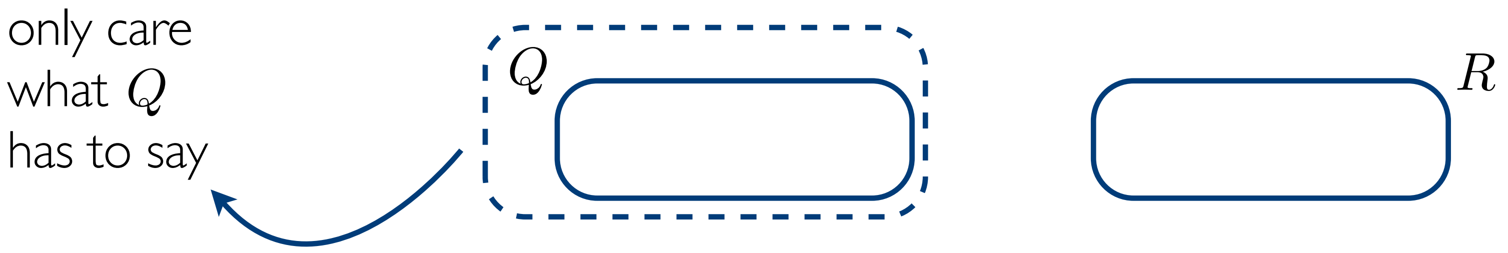
# Communicating ASP and the Polynomial Hierarchy

what is this talk about?

**this talk :** extension to multi-focused answer sets

*basic idea:* communicating programs are individual agents, where a single agent is the leader; one program is the main programs and the others are auxiliary programs (recursive idea!)

e.g. "I am only interested in what $Q$ has to say"




more generally: minimize over the information from $Q$

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## multi-focused answer sets (example)

example

office needs a new printer (stylish/dull ; silent/loud)

$E$ does not want one that is dull or loud

$M$ does not want one that loud

$B$ does not want one that is expensive (*i.e.* stylish and silent)

generate all 4 printer combinations, e.g. $P\!:\!dull, P\!:\!silent$

add additional rules (information) about undesirability, e.g.

$M\!:\!undesired \leftarrow P\!:\!loud$

$B\!:\!expensive \leftarrow P\!:\!stylish, P\!:\!silent$

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## multi-focused answer sets (example)

example (continued)

we have 4 communicating answer set programs

$$\{P{:}stylish, P{:}silent, B{:}expensive\}$$

$$\{P{:}stylish, P{:}loud, E{:}undesired, M{:}undesired\}$$

$$\{P{:}dull, P{:}silent, E{:}undesired\}$$

$$\{P{:}dull, P{:}loud, E{:}undesired, M{:}undesired\}$$

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## multi-focused answer sets (example)

example (continued)

we had 4 communicating answer set programs

~~$\{P:stylish, P:silent, B:expensive\}$~~

$\{P:stylish, P:loud, E:undesired, M:undesired\}$

$\{P:dull, P:silent, E:undesired\}$

$\{P:dull, P:loud, E:undesired, M:undesired\}$

$B$ is the boss; remove those communicating answer sets that are not subset minimal *w.r.t.* information from $B$

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## multi-focused answer sets (example)

example (continued)

we had 4 communicating answer set programs

~~$\{P:stylish, P:loud, E:undesired, M:undesired\}$~~

$\{P:dull, P:silent, E:undesired\}$

~~$\{P:dull, P:loud, E:undesired, M:undesired\}$~~

$M$ is the manager of $E$; remove those comm. answer sets that are not subset minimal *w.r.t.* information from $M$

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## multi-focused answer sets (example)

example (continued)

we had 4 communicating answer set programs, of which

$$\{P:dull, P:silent, E:undesired\}$$

is the unique $\{B, M(, E)\}$–focused answer set

the order in which we focus matters and affects the complexity

# Communicating ASP and the Polynomial Hierarchy

## semantics

a component program $Q$ is

- a (definite/normal/disjunctive) program ...
- that uses situated literals instead of normal literals.

a *communicating program* $\mathcal{P}$ is a set of component programs

for $\mathcal{P}$ a communicating program and $I$ a set of situated literals, the reduct $\mathcal{P}^I$ is defined as the set of reducts $Q^I$ of each component program $Q \in \mathcal{P}$.

# Communicating ASP and the Polynomial Hierarchy

## semantics (continued)

for $Q$ a component program and $I$ a set of situated literals, the reduct $Q^I$ is obtained by deleting:

- each rule with $not\ R\!:\!a$ and $R\!:\!a \in I$ ;
- each remaining literal of the form $not\ R\!:\!a$;
- each rule with $R\!:\!a$ such that
  - $R\!:\!a \notin I$ ; and
  - $R\!:\!a$ is not $Q$-local.

e.g. $Q\!:\!a \leftarrow R\!:\!b, Q\!:\!c$

in component program $Q$

not $Q$-local

$Q$-local situated literal

# Communicating ASP and the Polynomial Hierarchy

## semantics (continued)

for $Q$ a component program and $I$ a set of situated literals, the reduct $Q^I$ is obtained by deleting:

- each rule with $not\ R\!:\!a$ and $R\!:\!a \in I$ ;
- each remaining literal of the form $not\ R\!:\!a$;
- each rule with $R\!:\!a$ such that
  - $R\!:\!a \notin I$ ; and
  - $R\!:\!a$ is not $Q$-local.
- each remaining $R\!:\!a$ and $R\!:\!a$ is not $Q$-local.

# Communicating ASP and the Polynomial Hierarchy

## semantics (continued)

for $Q$ a component program and $I$ a set of situated literals, the reduct $Q^I$ is obtained by deleting:

1 - each rule with $not\ R\!:\!a$ and $R\!:\!a \in I$ ;
2 - each remaining literal of the form $not\ R\!:\!a$;
- each rule with $R\!:\!a$ such that
  - $R\!:\!a \notin I$ ; and
  - $R\!:\!a$ is not $Q$-local.
- each remaining $R\!:\!a$ and $R\!:\!a$ is not $Q$-local.

$I = \{Q\!:\!b, R\!:\!b\}$ 1

1 ~~$Q\!:\!a \leftarrow not\ Q\!:\!b$~~ $\quad R\!:\!a \leftarrow Q\!:\!a$

$Q\!:\!b \leftarrow$ ~~$not\ R\!:\!a$~~ 2 $\quad R\!:\!b \leftarrow Q\!:\!b$

# Communicating ASP and the Polynomial Hierarchy

## semantics (continued)

for $Q$ a component program and $I$ a set of situated literals, the reduct $Q^I$ is obtained by deleting:

1. each rule with $not\ R\!:\!a$ and $R\!:\!a \in I$ ;
2. each remaining literal of the form $not\ R\!:\!a$;
3. each rule with $R\!:\!a$ such that
   - $R\!:\!a \notin I$ ; and
   - $R\!:\!a$ is not $Q$-local.
4. each remaining $R\!:\!a$ and $R\!:\!a$ is not $Q$-local.

$I = \{Q\!:\!b, R\!:\!b\}$ (1)

(1) ~~$Q\!:\!a \leftarrow not\ Q\!:\!b$~~

$Q\!:\!b \leftarrow$ ~~$not\ R\!:\!a$~~ (2)

~~$R\!:\!a \leftarrow Q\!:\!a$~~ (3)

$R\!:\!b \leftarrow$ ~~$Q\!:\!b$~~ (4)

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## semantics (continued)

for $\mathcal{P}$ a communicating program and $\mathcal{P}^I$ the reduct, communicating answer sets are defined in the classical way

notice that all remaining programs are $Q$-local and thus correspond with a classical program for which we know the answer sets

a $(Q_1, Q_2, \ldots, Q_n)$-*focused answer set* $M$ of $\mathcal{P}$ is defined recursively as:

- $M$ is a $(Q_1, Q_2, \ldots, Q_{n-1})$-focused answer set of $\mathcal{P}$ and there is no other $(Q_1, Q_2, \ldots, Q_{n-1})$-focused answer set $M'$ with $M'_{\downarrow Q_n} \subset M_{\downarrow Q_n}$
- a ()-focused answer set is any comm. answer set of $\mathcal{P}$

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## expressiveness

with normal component programs

we can use ($Q_1$, $Q_2$, …, $Q_{n-1}$)-focused answer sets to verify whether a Quantified Boolean Formula of the form $\exists X_1 \forall X_2 \ldots \Theta X_n \cdot p(X_1, X_2, \ldots, X_n)$ is satisfiable

brave reasoning with ($Q_1$, $Q_2$, …, $Q_{n-1}$)-focused answer sets is $\Sigma_n^P$-hard
as it turns out, the task is $\Sigma_n^P$-complete

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## encoding example

encoding $\exists x \forall y \exists z \cdot (x \wedge y) \vee (\neg x \wedge y \wedge z) \vee (\neg x \wedge \neg y \wedge \neg z)$:

| | | | |
|---|---|---|---|
| (generate) | $Q_0 : x \leftarrow not\ \neg x$<br>$Q_0 : \neg x \leftarrow not\ x$ | $Q_0 : y \leftarrow not\ \neg y$<br>$Q_0 : \neg y \leftarrow not\ y$ | $Q_0 : z \leftarrow not\ \neg z$<br>$Q_0 : \neg z \leftarrow not\ z$ |
| (encode QBF) | $Q_0 : sat \leftarrow x, y$ | $Q_0 : sat \leftarrow \neg x, y, z$ | $Q_0 : sat \leftarrow \neg x, \neg y, \neg z$ |
| (technical) | $Q_0 : \neg sat \leftarrow not\ sat$ | | |
| (ensure $\exists z$) | $Q_1 : \neg x \leftarrow Q_0 : \neg x$<br>$Q_1 : x \leftarrow Q_0 : x$ | $Q_1 : y \leftarrow Q_0 : y$<br>$Q_1 : \neg y \leftarrow Q_0 : \neg y$ | $Q_1 : \neg sat \leftarrow Q_0 : \neg sat$ |
| (ensure $\forall y$) | $Q_2 : \neg x \leftarrow Q_0 : \neg x$<br>$Q_2 : x \leftarrow Q_0 : x$ | | $Q_2 : sat \leftarrow Q_0 : sat$ |

satisfiable iff $Q_0 : sat \in M$ with $M$ a $(Q_1, Q_2)$-focused answer set

# Communicating ASP and the Polynomial Hierarchy

## conclusion

conclusions:

- communication critically influences the expressiveness/complexity
  indeed, captures entire polynomial hierarchy
- idea is that of leaders and followers
  they can successively apply their preferences by eliminating answer sets
- on a technical level, multi-focused answer sets correspond with determining local minimality
- choice of communication mechanism is paramount w.r.t expressiveness of the overall system, irrespective of expressiveness of individual agents.

# Questions?