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One-and-only item recommendation with fuzzy logic techniques

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Abstract

Recommender systems anticipate users' needs by suggesting items that are likely to interest them. Most existing systems employ collaborative filtering (CF) techniques, searching for regularities in the way users have rated items. While in general a successful approach, CF cannot cope well with so-called one-and-only items, that is: items of which there is only one single instance (like an event), and which as such cannot be repetitively "sold". Typically such items are evaluated only after they have ceased being available, thereby thwarting the classical CF strategy. In this paper, we develop a conceptual framework for recommending one-and-only items. It uses fuzzy logic, which allows to reflect the graded/uncertain information in the domain, and to extend the CF paradigm, overcoming limitations of existing techniques. A possible application in the context of trade exhibition recommendation for e-government is discussed to illustrate the proposed conceptual framework.

Keywords: Recommender systems; Collaborative decision support; Fuzzy logic; Fuzzy relational calculus; Similarity

1. Introduction

With the advent of large-scale online applications, personalization has gained momentum as a means of challenging the information overload, as well as of understanding, and catering to, the needs of individuals or groups of customers. As a concrete example, personalization of e-government services is aimed at custom-tailoring the content government provides to the individual and business user. In many countries, e-government applications are growing rapidly and the amount of e-government websites, as well as the resources and services provided, are dynamically increasing. As a consequence, citizens may find it more and more difficult to locate relevant information from these websites. Matching particular citizens' and businesses' interests and needs is therefore one of the main challenges for e-government services, and intelligent decision support

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techniques are in demand to meet these personalization objectives. In this paper, specifically, we are concerned with recommender systems: applications that identify and suggest "interesting" items to a particular user-based on available information sources. The items range in substance from typical online-shopping products (books, movies, cosmetics, etc.) to more ephemeral goods, like recommendations for a patient to obtain a particular kind of medical care, or for a student to attend a given class, etc. Typical information sources include past purchase records, clickstream analysis, user profiles, explicit ratings of items, social network information (to synthesize user relationships), object hierarchies (to describe items' internal structure and their interrelationships), etc.

Various strategies have been proposed to put the information stream to use. Collaborative filtering (CF) infers user preferences for unseen items based on how "like-minded" individuals have evaluated those items. In content-based (CB) approaches, an orthogonal stance is taken, linking users with items similar to those they have liked in the past. Each of them comes with its particular drawbacks: CF has difficulties coping with sparse rating data, and cannot recommend an item which has not been evaluated yet by any user (the so-called "new item" problem). CB methods on the other hand tend to have their scope limited to the immediate neighbourhood of the user's past purchase or rating record; for instance, if a user is only known to have ordered a database book, the system will continue to only recommend database-related books, and not explore other interests of the user. This is in particular problematic for first-time customers (the "new user" problem).

Hybrid approaches combining different techniques, sometimes enhanced by knowledge-based methods, can overcome some of these limitations. In this paper, we develop a hybrid CF–CB conceptual framework in which we model user preferences, as well as user and item similarities, as fuzzy relations; the latter can deal with the graded nature of preferential information, and with the uncertainties inherent to the recommendation process. The rationale of our approach is then concisely summed up as "recommending future items if they are similar to past ones that similar users have liked". While being suitable for recommendation in general, this approach is particularly useful for "one-and-only" items, a problem which was first touched upon in [12]. For instance, a house put for sale is usually unique, and once sold is no longer eligible for recommendation; however, a judgment, by a related individual, of a house with similar characteristics conveys very useful information. Our method encompasses and generalizes the existing hybrid approaches of Guo and Lu [12,13] (based on item-based CF and semantic similarity) and of Perny and Zucker [21] (based on user-based CF and fuzzy similarity measures). Our conceptual framework also provides a way of presenting positive and negative feelings separately, and allows to deal with information shortage and information excess.

The remainder of this paper is organized as follows: in Section 2, we recall the necessary mathematical preliminaries on fuzzy relational calculus, and in Section 3 we briefly review related work on fuzzy recommender systems; in particular, we identify some of the drawbacks of existing approaches in light of the one-and-only item recommendation problem. The new conceptual framework and its associated algorithm are outlined in detail in Section 4. In Section 5, a simple illustration for the potential practical use of the framework is detailed in an e-government application concerning the recommendation of trade exhibitions to exporters. Finally, Section 6 concludes and points out further work.

2. Fuzzy relational calculus

Recall that a fuzzy set A in a universe U is a mapping from U to the unit interval [0,1], and that a fuzzy relation R from U to V is a fuzzy set in $U \times V$. Given a fuzzy relation R from U to V, its inverse relation R^{-1} is defined by $R^{-1}(v,u) = R(u,v)$. A fuzzy relation R from U to U is also called a fuzzy relation in U. It is reflexive if for all u in U, $R(u,u) = 1$ and symmetrical if $R(u,v) = R(v,u)$ for all u and v in U.

As a concrete example, we can define a fuzzy preference relation P from U, the universe of users, to I, the universe of items; for each user u and each item i, $P(u,i)$ denotes the degree to which i is preferred, or liked, by u.

Fuzzy relational calculus (see e.g. [16]) dictates the way information contained in fuzzy relations can be propagated, and in particular studies the various compositions of fuzzy relations. First recall that a triangular norm, or t-norm, $\mathscr{T}$ is a commutative, associative, increasing $[0,1]^2 \rightarrow [0,1]$ mapping that satisfies $\mathscr{T}(x,1) = x$ for all x in [0,1] and that an implicator $\mathscr{I}$ is a $[0,1]^2 \rightarrow [0,1]$ mapping with decreasing first and increasing second partial mappings that satisfies $\mathscr{I}(0,0) = \mathscr{I}(0,1) = \mathscr{I}(1,1) = 1$ and $\mathscr{I}(1,0) = 0$.

Definition 1 (***sup-$\mathcal{T}$-Composition***). Let R be a fuzzy relation from U to V, S a fuzzy relation from V to W, and $\mathcal{T}$ a t-norm. Then the sup-$\mathcal{T}$-composition $R \circ_{\mathcal{T}} S$ is a fuzzy relation from U to W defined by, for all u,w in $U \times W$,

$$R \circ_{\mathcal{T}} S(u,w) = \sup_{v \in V} \mathcal{T}(R(u,v), S(v,w)) \tag{1}$$

Definition 2 (***Subproduct***). Let R be a fuzzy relation from U to V, S a fuzzy relation from V to W, and $\mathcal{I}$ an implicator. Then the subproduct $R \lhd_{\mathcal{I}} S$ is a fuzzy relation from U to W defined by, for all u,w in $U \times W$,

$$R \lhd_{\mathcal{I}} S(u,w) = \inf_{v \in V} \mathcal{I}(R(u,v), S(v,w)) \tag{2}$$

The meaning of these compositions can be retraced by recalling the role of sup, inf, $\mathcal{T}$ and $\mathcal{I}$: (u,w) belongs to $R \circ_{\mathcal{T}} S$ to the extent that there exists **at least one** v in V such that u is related by R to v **and** v is related by S to w, and to $R \lhd_{\mathcal{I}} S$ insofar as for **every** v, **if** u is related to v by R, **then** v is related by S to w.

For example, let R denote a symmetrical fuzzy relation in U that expresses similarity between the users of a given e-service, and let P be the above-defined fuzzy preference relation. Then the sup-$\mathcal{T}$-composition $R \circ_{\mathcal{T}} P$ expresses for each couple (u,i) in $U \times I$, to what degree there exists a similar user to u who likes i; taking the subcomposition $R \lhd_{\mathcal{I}} P$ allows us to identify the items that are liked by all users that are similar to u.

For practical applications, the relational compositions may be too harsh because of the use of sup and inf. For instance, $R \lhd_{\mathcal{I}} S(u,w) = 0$ as soon as there exists a single v such that $R(u,v) = 1$ and $S(v,w) = 0$, regardless of the choice of $\mathcal{I}$. In such cases, it is worthwhile to mellow down the compositions (1) and (2) by taking an aggregated value, instead of simply the best or worst one. For this purpose, let be $\otimes$ be a fuzzy aggregation operator,[1] and define $R \circ_{\mathcal{T}}^{\otimes} S$ and $R \lhd_{\mathcal{I}}^{\otimes} S$ by

$$R \circ_{\mathcal{T}}^{\otimes} S(u,w) = \otimes_{v \in V} \mathcal{T}(R(u,v), S(v,w)) \tag{3}$$
$$R \lhd_{\mathcal{I}}^{\otimes} S(u,w) = \otimes_{v \in V} \mathcal{I}(R(u,v), S(v,w)) \tag{4}$$

Possible aggregation operators $\otimes$ include

- simple weighted average: $\otimes_{i=1}^{n} a_i = \frac{\sum_{i=1}^{n} c_i a_i}{\sum_{i=1}^{n} c_i}, \quad c_i > 0\ (i = 1,\ldots,n)$
- ordered weighted average (OWA, Yager [31]): $\otimes_{i=1}^{n} a_i = \sum_{j=1}^{n} c_j b_j$, $c_j \in [0,1]$, $\sum_{j=1}^{n} c_j = 1$, b_j is the jth-largest of the a_i ($j = 1,\ldots,n$).

3. Related work

3.1. Recommender systems

Karypis [15] defined a recommender system as a personalized information filtering technology, used to either predict whether a particular user will like a particular item, or to identify a set of items that will be of interest to a certain user.

Various explicit and implicit data collection methods have been used in recommender systems. Examples of explicit data collection methods include: asking a user to rate an item; to rank a collection of items; to choose the best one between two items; and to create a list of items that he/she likes. Implicit data collection is mainly achieved by keeping purchase records and analyzing clickstream data; e.g. the number of times users have accessed an item, or the time they have spent looking at them all convey useful information. To generate the recommendations, a set of algorithms has been developed to analyze the data and to derive a list of recommended items for the user. Most existing recommender systems adopt two types of techniques: content-based approaches and collaborative filtering approaches.

[1] In this paper, we follow Klir and Folger's [17] general definition of a fuzzy aggregation operator, that is: we only require $\otimes(0,\ldots 0) = 0$, $\otimes(1,\ldots 1) = 1$ and $\otimes$ is monotonically increasing in all its arguments.

A CB approach relies mainly on content and relevant profiles to offer recommendations. It recommends objects that are similar to those in which the user has been interested in the past. It originally derived from machine learning research and has been adopted by the information retrieval society, where textual documents are recommended by comparing their contents and user profiles [26]. More generally, CB approaches can be taken to encompass any recommendation scheme that involves an internal representation of the items (often by a vector of numerical values, as e.g. in the semantic product relevance model developed in [12,13] and described in Section 5). An inherent drawback of CB approaches is that they consider each individual's (explicit or implicit) preference record in isolation, thus turning a blind eye to the regularities identifiable from group behaviour.

CF is the most popular recommendation technique used in various different applications, such as the recommendation of web pages, movies, articles, etc. (see e.g. [14,24,29]). Both user-based and item-based techniques have been identified within the CF research community. User-based CF is implemented in two steps:

(1) A set of k nearest neighbours of an active user are computed. This is performed by computing correlations or similarities between user records and the active user;
(2) A prediction value is produced for the active user on unrated (or unseen) items, based on a combination of the ratings known from the nearest neighbours.

Two important problems with this approach are sparsity and lack of scalability. Regarding the second problem, item-based CF avoids the bottleneck in user relationship computations by first considering the relationships among items. Rather than finding user neighbours, the system attempts to find k similar items that are rated (or experienced) by different users in some similar way. Then, for a target item, predictions can be generated, for example, by taking a weighted average of the active user's item ratings (or weights) on these neighbour items. Thus, these algorithms alleviate the scalability problem that exists in user-based CF algorithms. Item-based CF has been shown to achieve prediction accuracies that are comparable to or even better than user-based CF algorithms [25]. However, item-based CF algorithms still suffer from the problems associated with data sparsity, and they still lack the ability to provide recommendations or predictions for new or recently added items.

In view of these limitations, a common thread in recommender system research has been the need to combine recommendation techniques in a hybrid approach to achieve peak performance. For instance, the combination of CF and CB techniques have been proposed by Balabanovic and Shoham [1], Good et al. [11], and Condliff et al. [7]. Also, other hybrid models exist in the literature, such as, Cho et al. [6] who combine CF and decision trees, Flesca et al. [9] who proposed a web page recommendation based on a combination of a user's browsing preference and website content, and Martín et al. [18] who used page categories [4] to overcome the new item problem.

3.2. Recommender systems with fuzzy logic techniques

Given the proliferance of rating and weighting schemes used in the existing approaches, topped by the fact that very often decisions have to be made under incomplete and/or conflicting information, fuzzy set theory lends itself well to the recommendation problem. Yager [33] devised a fuzzy logic-based conceptual framework for the representation and subsequent construction of justifications and recommendation rules. It uses an internal description of the items, and relies solely on the preferences of the target user. As such, the proposed recommendation strategies are purely content-based (or reclusive, as the author calls them). Carbo and Molina [5] developed a CF-based algorithm in which ratings and recommendations can be linguistic labels represented by fuzzy sets. Perny and Zucker [20,21] approached recommender systems from a decision support perspective, noting that such applications position themselves between the archetypical problems of individual and group decision making. They coined the term "Collaborative Decision Support" (CDS) to denote decision making problems where individuals seek recommendations for their personal choices, the other individuals being considered as possible advisors. In that light, they pursued a hybrid CB–CF approach that involves the following fuzzy relations:

- P^+, from U to I, expresses *positive feelings* (satisfaction) of a user about an item
- P^-, from U to I, expresses *negative feelings* (dissatisfaction) of a user about an item
- S, in I, expresses *similarity* between items
- R, in U, expresses *similarity* between users
- Q, from U to I, expresses the *qualification* of a user w.r.t. his rating of an item
- $\widehat{P}$, from U to I, expresses the *predicted preference* of a user for an item

P^+ and P^- are derived from the actual preference information (e.g., ratings) by a suitable transformation, where $\min(P^+(u,i), P^-(u,i)) = 0$ is imposed for each (u,i) in $U \times I$, indicating that a user u is either positively, or negatively, inclined about an item i. Using appropriate fuzzy similarity measures, for each item i, and each user u, a neighbourhood of k most similar elements is constructed and denoted $N_k(i)$, respectively $N_k(u)$; thanks to the use of neighbourhoods, the entire search space needs not be traversed in producing recommendations. Next, $Q(u,i)$ can be a self- or peer-evaluation of the confidence about u's rating of i, to strengthen or diminish its impact in the generation of recommendations. Finally, the target relation $\widehat{P}$ is computed by the following formula, for u in U and i in I:

$$\widehat{P}(u,i) = (1-\beta)\widehat{P}_{\mathrm{CB}}(u,i) + \beta\widehat{P}_{\mathrm{CF}}(u,i) \tag{5}$$

where β is a weighting parameter in [0,1] and $\widehat{P}_{\mathrm{CB}}(u,i)$ and $\widehat{P}_{\mathrm{CF}}(u,i)$ are the content-based, respectively collaborative filtering components to the recommendation, defined by, for a t-norm $\mathscr{T}$,

$$\widehat{P}_{\mathrm{CB}}(u,i) = \mathscr{T}(P^+_{\mathrm{CB}}(u,i), 1 - P^-_{\mathrm{CB}}(u,i)) \tag{6}$$

$$\widehat{P}_{\mathrm{CF}}(u,i) = \mathscr{T}(P^+_{\mathrm{CF}}(u,i), 1 - P^-_{\mathrm{CF}}(u,i)) \tag{7}$$

$$\widehat{P}^+_{\mathrm{CB}}(u,i) = \sup_{j \in N_k(i)} \mathscr{T}(P^+(u,j), S(j,i)) \tag{8}$$

$$\widehat{P}^-_{\mathrm{CB}}(u,i) = \sup_{j \in N_k(i)} \mathscr{T}(P^-(u,j), S(j,i)) \tag{9}$$

$$\widehat{P}^+_{\mathrm{CF}}(u,i) = \sup_{v \in N_k(u)} \mathscr{T}(\mathscr{T}(Q(v,i), P^+(v,i)), R(v,u)) \tag{10}$$

$$\widehat{P}^-_{\mathrm{CF}}(u,i) = \sup\_v \in N_k(u) \mathscr{T}(\mathscr{T}(Q(v,i), P^-(v,i)), R(v,u)) \tag{11}$$

with these definitions, user u receives a high recommendation on item i if

- i is similar to any j_1 which u likes and is not similar to any j_2 which u dislikes {formulas (6) and (8,9)}.
- i is liked by a given v_1 who is similar to u, and there does not exist any v_2 similar to u who dislikes i; the appearance of $Q(v,i)$ in the formulas is to ensure the authority of v's evaluation of i {formulas (7) and (10,11)}.

Noting that the occurence of a single positive or negative evaluation in (8)–(11) can affect the final outcome dramatically, Perny and Zucker proposed mellowed versions of these formulas akin to the aggregated compositions (3) and (4). They furthermore fine-tuned and refined their recommendation algorithm by thresholding and machine learning techniques, and implemented it in the "Film-Conseil" movie recommender system.

Perny and Zucker's conceptual framework, while quite elaborate and flexible in itself, has some important drawbacks:

- It is hard to set an appropriate value for the parameter β balancing the impact of CB and CF contributions to the final recommendation $\widehat{P}(u,i)$.
- The CF component is useless for recommending new items; that is, for such items, their approach is totally reliant on pure CB recommendation.

4. A hybrid fuzzy logic based recommendation framework

With the strengths and weaknesses of the various above-described approaches in mind, in this section we develop a conceptual framework for item recommendation that performs well for one-and-only items, and that can exploit the full potential of both CB and CF paradigms. Our proposal can be formulated as an extension to the seminal work of Perny and Zucker. We therefore maintain the symbols these authors introduced to denote the various fuzzy relations. Furthermore, we will not use the qualification relation Q, which can in fact be easily absorbed into the definitions of P^+ and P^- by putting

$$P^+(u,i) := \mathscr{T}(P^+(u,i), Q(u,i)) \tag{12}$$

$$P^-(u,i) := \mathscr{T}(P^-(u,i), Q(u,i)) \tag{13}$$

with $\mathscr{T}$ a t-norm. The description of the conceptual framework is split into four phases:

(1) modelling users' preferences;
(2) computing similarity between items;
(3) computing similarity (or influence) between users;
(4) generating recommendations to users.

4.1. Preference modelling

Note that the restriction in [21] that $\min(P^+(u,i),P^-(u,i)) = 0$ splits up users into crisp "pro" and "contra" sides w.r.t. a given item; in other words, a user cannot feel slightly positive and slightly negative about an item at the same time. This restriction appears unnatural, because generally there is a smooth transition between the two camps, with many users exhibiting both positive and negative attitudes towards (different aspects of) an item. Moreover, there is often an element of uncertainty and/or conflict in our assessment of users' preferences. In the context of multiple-criteria decision making, this was also noted by Fortemps and Słowinski [10], and later by Arieli et al. [8], who used independent scales to handle graded relevance of positive and negative arguments provided in preferential information. In accordance with this approach, we allow any $(P^+(u,i), P^-(u,i))$ in $[0,1]^2$ to express a user's feelings w.r.t. an item. The following "special" values can be distinguished:

(1,0)$\rightarrow$ Outspoken preference.
(0,1)$\rightarrow$ Outspoken dislike.
(0,0)$\rightarrow$ Ignorance. u has not experienced, or expressed any opinion about, i.
(1,1)$\rightarrow$ Conflict. u has expressed contradicting opinions about i.

It is important to stress that these two-sided evaluations are *cognitive* states reflecting the algorithm's *knowledge* about the user's preferences; each component captures the evidence gathered by the algorithm through explicit or implicit user querying about the item.

To quantify the amount, and the quality, of the information we have about the evaluation of i by u, we introduce the [0,1]-valued measure $\mathscr{K}$ ("knowledge"), given by

$$\mathscr{K}(u,i) = 1 - |P^+(u,i) + P^-(u,i) - 1|$$

Then $\mathscr{K}(u,i) = 0$ if $(P^+(u,i), P^-(u,i)) = (0,0)$ or $(P^+(u,i), P^-(u,i)) = (1,1)$, $\mathscr{K}(u,i) = 1$ if $P^+(u,i) + P^-(u,i) = 1$, and $0 < \mathscr{K}(u,i) < 1$ in all other situations. $\mathscr{K}$ will be used later on as a measure of the usefulness of a user's evaluation of an item for the recommendation process.

4.2. Item similarity

As mentioned in Section 3, there are two basic ways to compute how similar, or close, two items are. The first one derives from the CB paradigm, and requires an explicit representation of an item in terms of descriptive attributes. It is independent of the availability of user ratings. The second one is obtained by item-based

CF, and only requires that the items have received sufficient ratings, while no internal description of the items is needed.

CB Similarity. Computing [0,1]-valued similarity between items described by attribute vectors is a problem well-covered in the literature on fuzzy set theory, see e.g. [2]. Outside the fuzzy community, the concept of semantic similarity among objects and classes has attracted considerable attention during the past few years [12,13,19,22]. Semantic information about an item consists of its attributes, relationships to other items, and other meta-information. Various [0,1]-valued measures have been developed for quantifying the notion of semantic similarity. In general, we assume that CB similarity is represented by a fuzzy relation S_{CB} in I which is at least reflexive and symmetrical.

Item-Based CF Similarity. This type of similarity compares items by looking at the ratings each user has given them. Past research [3] has pointed out that this kind of similarity is best evaluated using Pearson's correlation coefficient, that is, for two items i and j in I we compute[2]

$$S_{\mathrm{CF}}(i,j) = \frac{\sum\limits_{u\in U}(P^+(u,i) - \overline{P_i^+}).(P^+(u,j) - \overline{P_j^+})}{\sqrt{\sum\limits_{u\in U}(P^+(u,i) - \overline{P_i^+})^2.\sum\limits_{u\in U}(P^+(u,i) - \overline{P_i^+})^2}} \tag{14}$$

where $\overline{P_i^+}$ and $\overline{P_j^+}$ represent the average rating value of different users on item i and j, respectively.

Integrating CB and Item-Based CF Similarity. CB similarity is especially useful for information-rich items: as the items' description gets finer and more accurate, a measure of similarity will have more and more discriminating power. On the other hand, CF similarity can benefit from statistical features present in the dataset. Note that it can still be useful in the context of one-and-only items, as it may help identify relationships between previously rated items; this – as will be discussed in the next paragraphs – is useful for computing user similarity and producing recommendations. The fuzzy relation S describing item similarity is then given by

$$S(i,j) = \otimes(S_{\mathrm{CB}}(i,j), S_{\mathrm{CF}}(i,j)) \tag{15}$$

with $\otimes$ an aggregation operator. In case either S_{CB} or S_{CF} is not available, $\otimes$ can be simply defined by $\otimes(x,y) = x$ or $\otimes(x,y) = y$. It is easy to verify that, regardless of $\otimes$, S is always a reflexive and symmetric relation in I.

4.3. User similarity

User similarity is more complicated. While it is technically possible to pursue the same approach as with CB item similarity by comparing user description (profile) vectors, this is often impractical because in many e-services users are quasi anonymous. Moreover, the relation R we are looking for is not necessarily symmetrical, bearing in mind that in user-based CF the role of fellow users is primarily as advisors who can direct the target user to interesting items.

Guided by these arguments, Perny and Zucker [21] considered, amongst others, "positive influence" of u over v, by evaluating to what degree everything u (dis)likes, v (dis)likes too. This amounts to the following formula:

$$R(u,v) = \mathscr{T}\left(\inf_{i\in I}\mathscr{I}(P^+(u,i),P^+(v,i)), \inf_{i\in I}\mathscr{I}(P^-(u,i),P^-(v,i))\right) \tag{16}$$

$$= \mathscr{T}((P^+\lhd_{\mathscr{I}}(P^+)^{-1})(u,v),(P^-\lhd_{\mathscr{I}}(P^-)^{-1})(u,v)) \tag{17}$$

where $\mathscr{T}$ is a t-norm and $\mathscr{I}$ is an implicator. Note that $R(u,u) = 1$ as soon as $\mathscr{I}$ satisfies $\mathscr{I}(x,x) = 1$. This is true for many implicators, Łukasiewicz' implicator $\mathscr{I}_{\mathrm{W}}$ in particular satisfies it as $I_{\mathrm{W}}(x,x) = \min(1,1 - x + x) = 1$.

An important disadvantage of this formula (and of user-based CF in general) is that by focussing strictly on those items that both u and v have rated in common, it tends to overlook a lot of interesting relationships

[2] A similar analysis can be made for P^-, and both results can be aggregated.

existing in the domain. As an example, suppose that I is a music collection. Given that user u gave a positive evaluation for CDs by Eminem and 50 Cent, and that v likes Ice-T and P. Diddy (but did not rate any of the items u bought), hence both of them seem to be fond of rap music, and u could be an excellent advisor for v. However, since they have no rated items in common, the CF algorithm cannot discover this shared interest.

To circumvent this problem, we can reformulate the user similarity criterion as "for every item i that u likes, there exists a *similar* item that v likes". In other words, we are using item similarity to obtain a more accurate assessment of user relationships. This means we have to evaluate the sup-$\mathcal{T}$-compositions $S\circ_{\mathcal{T}}(P^{+})^{-1}$ and $S\circ_{\mathcal{T}}(P^{-})^{-1}$ instead of $(P^{+})^{-1}$ and $(P^{-})^{-1}$, and the formulas (16) and (17) can be replaced by

$$R(u,v)=\mathcal{T}\left(\inf_{i\in I}\mathcal{I}(P^{+}(u,i),\sup_{j\in I}\mathcal{T}(S(i,j),P^{+}(v,j))),\inf_{i\in I}\mathcal{I}(P^{-}(u,i),\sup_{j\in I}\mathcal{T}(S(i,j),P^{-}(v,j)))\right) \tag{18}$$

$$=\mathcal{T}((P^{+}\lhd_{\mathcal{I}}(S\circ_{\mathcal{T}}(P^{+})^{-1}))(u,v),(P^{-}\lhd_{\mathcal{I}}(S\circ_{\mathcal{T}}(P^{-})^{-1}))(u,v)) \tag{19}$$

Proposition 1. *For every t-norm $\mathcal{T}$, implicator $\mathcal{I}$ and elements u and v in U, the value of (19) is at least as high as that of (17).*

Proof. Since we assumed that $S(i,i) = 1$ for every i in I, we find

$$\begin{aligned}R(u,v)&=\mathcal{T}\Bigg(\inf_{i\in I}\mathcal{I}(P^{+}(u,i),\sup_{j\in I}\mathcal{T}(S(i,j),P^{+}(v,j))),\\&\qquad\inf_{i\in I}\mathcal{I}(P^{-}(u,i),\sup_{j\in I}\mathcal{T}(S(i,j),P^{-}(v,j)))\Bigg)\\&\geqslant\mathcal{T}\Bigg(\inf_{i\in I}\mathcal{I}(P^{+}(u,i),\mathcal{T}(S(i,i),P^{+}(v,i))),\\&\qquad\inf_{i\in I}\mathcal{I}(P^{-}(u,i),\mathcal{T}(S(i,i),P^{-}(v,i)))\Bigg)\\&=\mathcal{T}\left(\inf_{i\in I}\mathcal{I}(P^{+}(u,i),P^{+}(v,i)),\inf_{i\in I}\mathcal{I}(P^{-}(u,i),P^{-}(v,i))\right)\qquad\square\end{aligned}$$

Proposition 1 shows that any link discovered through classical user-based CF, will also be found with our approach, so (19) is truly an extension of (17). We can further custom-tailor this formula by replacing the harsh subcomposition inside it by an aggregation-style composition, that is:

$$\begin{aligned}R(u,v)&=\mathcal{T}\Bigg(\otimes_{i\in I}\mathcal{I}(P^{+}(u,i),up_{j\in I}\mathcal{T}(S(i,j),P^{+}(v,j))),\\&\qquad\otimes_{i\in I}\mathcal{I}(P^{-}(u,i),\sup_{j\in I}\mathcal{T}(S(i,j),P^{-}(v,j)))\Bigg)\end{aligned}\tag{20}$$

$$\begin{aligned}&=\mathcal{T}\Big((P^{+}\lhd_{\mathcal{I}}^{\otimes}(S\circ_{\mathcal{T}}(P^{+})^{-1}))(u,v),\\&\qquad(P^{-}\lhd_{\mathcal{I}}^{\otimes}(S\circ_{\mathcal{T}}(P^{-})^{-1}))(u,v)\Big)\end{aligned}\tag{21}$$

Remark 1. There exist several viable options for the choice of the aggregation operator $\otimes$ in formula (21). We give two illustrative examples, referring to weighted averages and OWA operators, respectively:

(1) Using simple weighted average aggregation, item weights c_i can be chosen to reflect the importance of each item in the overall user similarity evaluation. A suitable guideline for determining weights is the following: if i, nor anything remotely similar to i, was rated by v, this item should not have an impact on $R(u,v)$. We can obtain this behaviour by putting

$$c_i=\sup_{j\in I}\mathcal{T}(S(i,j),\mathcal{K}(v,j)) \tag{22}$$

with $\mathcal{T}$ a t-norm.

(2) We can also use Yager's method from [32] for defining OWA operators from fuzzy quantifiers.[3] Given a fuzzy quantifier Q, OWA weights can be defined by $c_j = Q(\frac{j}{n}) - Q(\frac{j-1}{n})$ for $j = 1, \ldots, n$. In [33], for instance, $Q_1(x) = x$ and $Q_2(x) = x^2$ were proposed, with the associated interpretations of "some" and "most", respectively. When used in (21), the corresponding OWA operators can be used to evaluate whether for some, or most, of the items that u (dis)likes, there exists a similar item that v (dis)likes.

Remark 2. For efficiency purposes, it is better to replace I in (20) by $N_k(i)$ so that only a close neighbourhood of each item i is considered during user similarity evaluation. Note that distant items j cannot make a substantial contribution to the outcome anyway because of the low value of $S(i,j)$. Another simple but effective optimization is to take the similarity neighbourhood of item i in (20) into account, only when i itself has not been rated by v. Consequently, bottleneck problems in dense databases are avoided, while for sparse databases optimal exploitation of the available information is guaranteed.

4.4. Generating recommendations

The same technique that we have used to extend the user similarity assessment problem by looking at items *similar* to those a user has rated, can also be applied to recommendation itself, that is, to the prediction of the positive and negative evaluation values of user u for an item i that he has not yet seen or rated. Combining the information that the fuzzy relations R and S provide, we replace Perny and Zucker's recommendation formulas (6)–(11) by the following ones:

$$\widehat{P}(u,i) = \left(\widehat{P}^{+}(u,i), 1 - \widehat{P}^{-}(u,i)\right) \tag{23}$$

$$\widehat{P}^{+}(u,i) = \sup_{v \in N_k(u)} \mathscr{T}\left(\sup_{j \in N_k(i)} \mathscr{T}(S(i,j), P^{+}(v,j)), R(v,u)\right) \tag{24}$$

$$\widehat{P}^{-}(u,i) = \sup_{v \in N_k(u)} \mathscr{T}\left(\sup_{j \in N_k(i)} \mathscr{T}(S(i,j), P^{-}(v,j)), R(v,u)\right) \tag{25}$$

Proposition 2. *For every t-norm $\mathscr{T}$ and every elements u in U and i in I, the value of (24) is at least as high as that of (8) and (10), and the value of (25) is at least as high as that of (9) and (11).*

Proof. Similar to that of Proposition 1. □

Proposition 2 implies that formulas (24) and (25) encompass, at the same time, CB and CF paradigms. This can also be seen by the following:

(1) If we replace the supremum over all v in U by u in (24), then we obtain

$$\begin{aligned}\widehat{P}^{+}(u,i) &= \mathscr{T}\left(\sup_{j \in N_k(i)} \mathscr{T}(S(i,j), P^{+}(u,j)), R(u,u)\right) \\ &= \mathscr{T}\left(\sup_{j \in N_k(i)} \mathscr{T}(S(i,j), P^{+}(u,j)), 1\right) \\ &= \sup_{j \in N_k(i)} \mathscr{T}(S(i,j), P^{+}(u,j))\end{aligned} \tag{26}$$

or Perny and Zucker's pure content-based result.

[3] For definiteness, by a fuzzy quantifier we mean a fuzzy set Q in [0,1] such that $Q(0) = 0$, $Q(1) = 1$ and $Q(x) \leqslant Q(y)$ whenever $x \leqslant y$. Fuzzy quantifiers generalize the existential and universal quantifiers from classical logic.

(2) If we replace the supremum over all j in I by i in (24), then we obtain

$$\begin{aligned}\widehat{P}^{+}(u,i) &= \sup_{v \in N_k(u)} \mathcal{T}(\mathcal{T}(S(i,i),P^{+}(v,i)),R(v,u)) \\ &= \sup_{v \in N_k(u)} \mathcal{T}(\mathcal{T}(1,P^{+}(v,i)),R(v,u)) \\ &= \sup_{v \in N_k(u)} \mathcal{T}(P^{+}(v,i),R(v,u))\end{aligned} \tag{27}$$

or traditional user-based collaborative filtering.

As a byproduct of our approach, the need for choosing an appropriate β value to balance between the two components vanishes, because whichever of them has the stronger impact prevails. Moreover, these formulas allow for recommendations that the existing hybrid approach could never come up with: if u is similar to v, and v likes j which is similar to i, then i can be considered interesting for u. Not only does this idea allow the algorithm to explore new regions in the search space, it is in particular relevant to one-and-only item recommendation, for which the classical CF formula (7) does not yield any result. As evidenced by (24,25), in our setting users *can* influence each other even in this special situation.

Remark 3. Just like before, these formulas can be straightforwardly replaced by aggregated compositions. Note that in all cases the final outcome (23) of the algorithm is two-valued: $\widehat{P}(u,i)$ juxtaposes the arguments in favour of, respectively against, recommending u to i. If necessary, a suitable transformation to the linearly ordered scale [0,1] can be picked; this is the case, for example, if a top-N recommendation is to be obtained from it, that is: a set of N most attractive items among the (unseen) elements of I for a particular user has to be constructed. Simply using a t-norm as in formula (5), or an average value, might have some adverse effects,

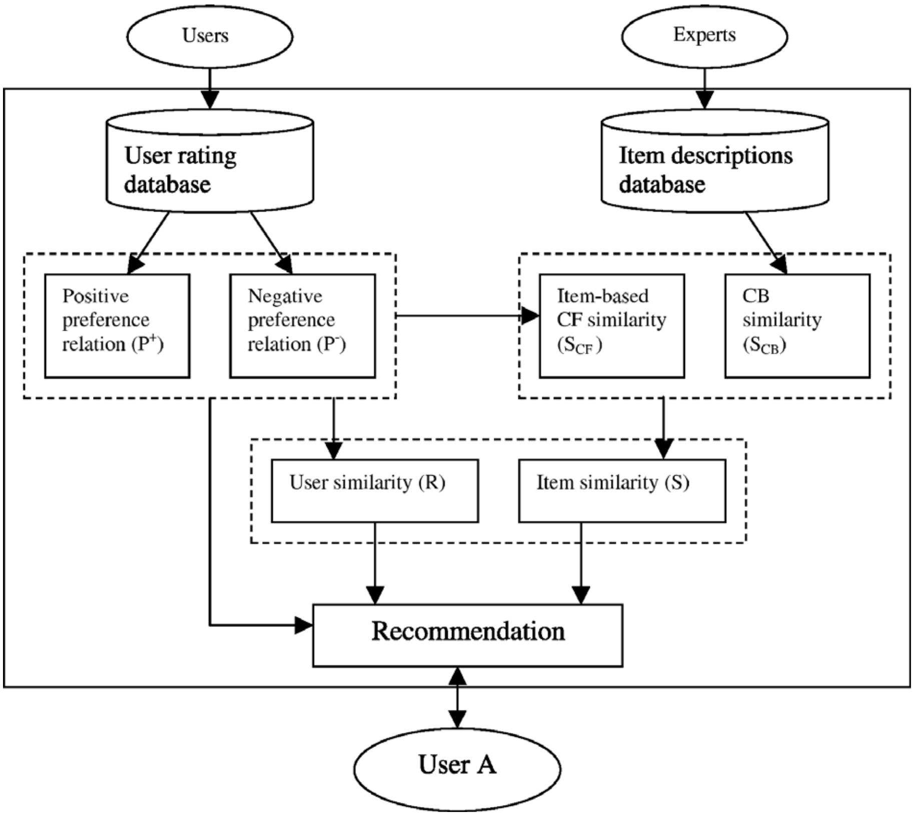

Fig. 1. Hybrid fuzzy recommendation framework.

because it disregards the amount of information (shortage or excess) that the algorithm has been able to gather about this particular user and item. In [10], some useful and more elaborate scoring procedures, tailored to $[0,1]^2$ positive/negative preference evaluations, are proposed. Otherwise, some common-sense "fuzzy" reasoning may be applied, such as to increase the eligibility of an item for recommendation as its $\widehat{P}^+$ component gets much greater than its $\widehat{P}^-$ component.

A schematic rendering of our conceptual framework is depicted in Fig. 1. Although a complex mathematical apparatus is underneath, the general structure is easy-to-grasp. Certain of its features may be switched on or off according to what the application at hand requires. For instance, if symmetry between positive and negative arguments is assumed (usually a simplifying assumption in real life), the algorithms can proceed with only the fuzzy preference relation P^+. If insufficient ratings have been recorded, and items come with a rich internal description, there may be no need for the item-based CF feature, etc.

In terms of complexity, our framework's increased search ability and expressiveness comes with an extra cost, as the construction of R and $\widehat{P}$ require the computation of an additional supremum for any couple of elements, yet this cost can be kept within bounds by dynamically adapting the neighbourhood size according to the need for extra information.

5. A trade exhibition recommender system for e-government

The fifth annual Accenture e-government Study [23] indicates that personalization in e-government is emerging. E-government service is defined as using web-based information technologies to enhance government information services, and to enable citizens and businesses to make online transactions [30]. Here we consider a particular government-to-business personalization application concerned with the recommendation of international trade exhibitions. These events are frequently used in exporting firms' marketing strategies and are of great value for exporting firms to communicate with potential and current customers from many countries in a short period of time. As the number of trade exhibitions' visitors increases, trade exhibitions have become a relatively important promotion tool for industrial firms. It is vital that government help companies choosing the right trade exhibitions. However, in the context of Australia, Austrade[4] can only offer simple, one-fits-all database matching functions via the web-based e-government services channel for providing advices about international trade fairs. New techniques are needed that can satisfy interests and needs of particular business users.

Trade exhibitions are typical examples of one-and-only items: business representatives do not submit their ratings until well after the event has finished, so at each given instance in time there is a clear distinction between past, non-recommendable, rated items and future, recommendable, unrated items. In the remainder of this section, a small example is given to illustrate, conceptually, how our framework may overcome the difficulties in this domain.

First, we need to elicit users' (i.e. businesses') ratings of past trade events, which means we have to construct the fuzzy relations P^+ and P^-. For simplicity, we consider single-value numerical ratings on an integer scale from 1 (worst) to 5 (best). Assume that $p_{u,i}$ is the rating value of user u for item i; we put $p_{u,i} = 0$ if no rating is available for this user-item pair. To obtain the values of $P^+(u,i)$ and $P^-(u,i)$ for our algorithm we apply the following transformation:

$$P^+(u,i) = \begin{cases} \frac{p_{u,i}-1}{4} & \text{if } p_{u,i} \in \{1,2,3,4,5\} \\ 0 & \text{otherwise} \end{cases} \tag{28}$$

$$P^-(u,i) = \begin{cases} \frac{5-p_{u,i}}{4} & \text{if } p_{u,i} \in \{1,2,3,4,5\} \\ 0 & \text{otherwise} \end{cases} \tag{29}$$

[4] Austrade (http://www.austrade.gov.au), the Australian Trade Commission, is a government agency which helps companies explore the international market. Organizing international trade fair participation and helping more Australian companies succeed in export are the most important tasks conducted by Austrade.

Note that under these assumptions $\mathscr{K}(u,i)=1$ when u has rated i, and 0 otherwise. For illustration purposes, let us consider a small hypothetical database containing six past trade exhibitions (labelled i_1 through i_6) and two future ones (i_7 and i_8), as well as three users who have each rated some of the past trade exhibitions. Table 1 shows the ratings given by users, where as mentioned a 0 means no rating is available for this item. Using (28) and (29), the fuzzy relations P^+ and P^- are established and shown in Table 2.

The rating data in Table 2 could be used to establish one kind of item similarity, namely item-based CF similarity (cfr. formula (14)); however, since there are so few users, its computation is not very meaningful in this example, so we omit it here. Instead, we concentrate on CB similarity.

Regarding the internal structure of our items, in [12,13], a semantic product relevance (SPR) model has been introduced to represent the internal structure of trade exhibitions. In a nutshell, the SPR model allows domain experts to provide a degree (or weight) of relevance of a given target product to various product categories represented by a hierarchical concept tree. For example, a cosmetics exporter wants to find trade exhibition information about cosmetics. Domain experts can define a range of trade fairs related to it according to the product taxonomy, such as "health", "chemical", "beauty", "fashion" and "gift". Fig. 2 shows an example (again hypothetical) of the SPR model, with weights taken from an integer 1–5 scale. By repeating the process for different target products, there emerges a complex and detailed description of the trade exhibition by a vector of numerical relevance degrees. To obtain the fuzzy relation $S=S_{\mathrm{CB}}$ of our conceptual framework, we can use any [0,1]-valued similarity measure, e.g. the standard vector-based cosine similarity. For any two items i and j,

$$S(i,j)=\frac{\sum_{t=1}^{M}a_{t,i}\times a_{t,j}}{\sqrt{\sum_{t=1}^{M}(a_{t,i})^2}\sqrt{\sum_{t=1}^{M}(a_{t,j})^2}} \tag{30}$$

where $a_{t,i}$ and $a_{t,j}$ represent the relevance vectors of target product t to item i and j, respectively ($t=1,\ldots M$). It is clear that the resulting fuzzy relation S is indeed reflexive and symmetrical. For conscienceness, we omit the details of the item similarity calculation and just show its result in Table 3.

Our next task is to find out the relationships between users. For clarity of exposition and because of the small size of the example, we will not consider neighbourhoods or aggregated compositions here. To evaluate formula (19), we first compute the sup-$\mathscr{T}$-compositions $S\circ_{\mathscr{T}}(P^+)^{-1}$ and $S\circ_{\mathscr{T}}(P^-)^{-1}$ using e.g. $\mathscr{T}=\min$. For instance,

$$\begin{aligned} S\circ_{\mathscr{T}}(P^+)^{-1}(i_1,u_1) &= \sup_{j\in I}\mathscr{T}(S(i_1,j),(P^+)^{-1}(j,u_1)) \\ &= \max_{j=1}^{8}\mathscr{T}(S(i_1,j),P^+(u_1,j)) \\ &= \max(0,\mathscr{T}(1,0.85),0,\mathscr{T}(0.75,0.75),0,0,0,0)=0.85, \end{aligned}$$

Table 1
Integer ratings expressed by three users for the considered trade exhibitions

	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
u_1	0	5	1	4	0	0	0	0
u_2	5	0	0	3	0	1	0	0
u_3	0	5	0	0	1	0	0	0

Table 2
Fuzzy relations P^+ and P^-

P^+	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
u_1	0	1	0	0.75	0	0	0	0
u_2	1	0	0	0.5	0	0	0	0
u_3	0	1	0	0	0	0	0	0
P^-	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
u_1	0	0	1	0.25	0	0	0	0
u_2	0	0	0	0.5	0	1	0	0
u_3	0	0	0	0	1	0	0	0

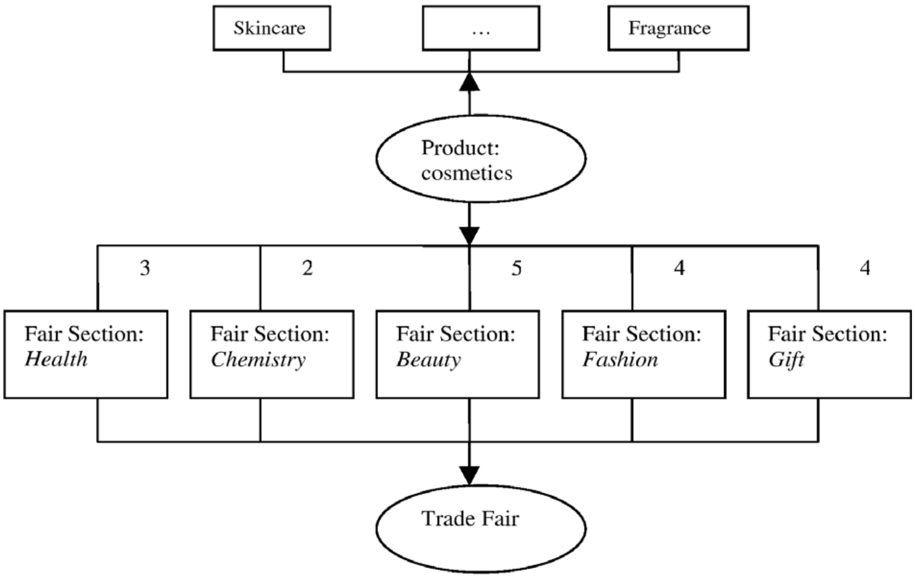

Fig. 2. An example of the SPR model.

Table 3
Fuzzy relation S: similarities between eight considered trade exhibitions

S	i_1	i_2	i_3	i_4	i_5	i_6	i_7	i_8
i_1	1.00	0.85	0.2	0.75	0.1	0	0.9	0
i_2		1.00	0.25	0.05	0	0.3	0	0.1
i_3			1.00	0	0.1	0.9	0	1
i_4				1.00	0.95	0.3	0.1	0.35
i_5					1.00	0.15	0.8	0
i_6						1.00	0.25	0.1
i_7							1.00	0
i_8								1.00

meaning that u_1 is known to like something similar to i_1 at least to degree 0.85. Both fuzzy relations are given in Table 4.

Using e.g. $\mathscr{I} = \mathscr{I}_{T_W}$, we can now compute R. For instance:

$$\begin{aligned}
R(u_1, u_2) &= \min\left(\min_{i=1}^{8} \mathscr{I}(P^+(u_1, i), S \circ_{\mathscr{T}} (P^+)^{-1}(i, u_2)),\right. \\
&\qquad \left.\min_{i=1}^{8} \mathscr{I}(P^-(u_1, i), S \circ_{\mathscr{T}} (P^-)^{-1}(i, u_2))\right) \\
&= \min(\min(\mathscr{I}(1, 0.85), \mathscr{I}(0.75, 0.75)), \min(\mathscr{I}(1, 0.9), \mathscr{I}(0.25, 0.5)) \\
&= 0.85,
\end{aligned}$$

revealing that u_1 is a reasonably good advisor for u_2, which is in accordance with the information we have. Table 5 lists all membership degrees for R.

Remark 4. Note that using Perny and Zucker's approach, formula (16) cannot identify the above-mentioned relationship between u_1 and u_2 because, for example, the preference of u_1 for i_2 is not matched by a rating of u_2 for i_2. In [20,21], a patch for this problem is proposed by only considering items commonly rated by u_1 and u_2. This, however, can have adverse effects, because it would imply that $R(u_1,u_3) = 1$, since both of them gave a top rating to i_2 and they have no other rated items in common. However, this attitude neglects the fact that u_3 does not like i_5, which is very similar to i_4 and which received a high rating by u_1. In general, this problem gets worse as the rating data become sparser.

Table 4
Fuzzy relations $S\circ_{\mathscr{T}}(P^+)^{-1}$ and $S\circ_{\mathscr{T}}(P^-)^{-1}$

$S\circ_{\mathscr{T}}(P^+)^{-1}$	u_1	u_2	u_3
i_1	0.85	1	0.85
i_2	1	0.85	1
i_3	0.25	0.2	0.25
i_4	0.75	0.75	0.05
i_5	0.7	0.5	0
i_6	0.3	0.3	0.3
i_7	0.1	0.9	0
i_8	0.35	0.35	0.1
$S\circ_{\mathscr{T}}(P^-)^{-1}$	u_1	u_2	u_3
i_1	0.25	0.5	0.1
i_2	0.25	0.3	0
i_3	1	0.9	0.1
i_4	0.25	0.5	0.95
i_5	0.25	0.5	1
i_6	0.9	1	0.15
i_7	0.1	0.25	0.8
i_8	1	0.35	0

Table 5
Fuzzy relation R

R	u_1	u_2	u_3
u_1	1	0.85	0.1
u_2	0.75	1	0.15
u_3	0.25	0.5	1

Assume now that we want to find out whether or not to recommend i_7 and i_8 to any of the users. We apply formulas (23)–(25) and obtain for e.g. u_1 and i_7 (still using $\mathscr{T} = \min$):

$$\begin{aligned}
\widehat{P}^+(u_1, i_7) &= \max_{v=1}^{3} \mathscr{T}(S\circ_{\mathscr{T}}(P^+)^{-1}(i_7, v), R(v, u_1)) \\
&= \max(\min(0.1, 1), \min(0.9, 0.75), \min(0, 0.25)) = 0.75 \\
\widehat{P}^-(u_1, i_7) &= \max_{v=1}^{3} \mathscr{T}(S\circ_{\mathscr{T}}(P^-)^{-1}(i_7, v), R(v, u_1)) \\
&= \max(\min(0.1, 1), \min(0.25, 0.75), \min(0.8, 0.25)) = 0.25 \\
\widehat{P}(u_1, i_7) &= (0.75, 0.25).
\end{aligned}$$

The results are summarized in Table 6. Note that these values are indeed intuitive; i_7 is considered attractive to u_2 because of its similarity to i_1, and to u_1 because this user is influenced by u_2; note that Perny and Zucker's approach would be unable to infer the latter because the data fit neither formula (6) nor (7). For quite

Table 6
Results of the recommendation process: fuzzy relations $\widehat{P}^+$ and $\widehat{P}^-$

$\widehat{P}^+$	i_7	i_8
u_1	0.75	0.35
u_2	0.9	0.35
u_3	0.15	0.15
$\widehat{P}^-$	i_7	i_8
u_1	0.25	1
u_2	0.5	0.85
u_3	0.8	0.15

analogous reasons, exhibition i_8 is unlikely to interest either of u_1 and u_2, while i_7 is not recommendable to u_3. The algorithm produces an indecisive result for (u_3,i_8) because there is no really convincing evidence to cut the knot for this user-item couple; this is reflected by the fact that $\mathscr{K}(\widehat{P}(u_3,i_8)) = 0.15 + 0.15 = 0.3$. Such uncertainty can be usefully taken into account when producing final recommendations, in a sense that if we had to choose between recommending either i_7 or i_8 to u_3, i_8 would be the one to pick as there is no clear argument against that alternative. Remark on the other hand that using formula (5), both $\widehat{P}(u_3,i_7)$ and $\widehat{P}(u_3,i_8)$ would be 0 which wrongly portrays both items as equally undesirable.

6. Conclusion

This paper has introduced a new conceptual framework for the recommendation, in the context of e-services, of items about which we only assumed that a sufficiently rich internal representation is available. The most important qualities of our results are:

(1) The use of fuzzy relations and their various compositions allows to capture and exploit the relationships between users and items existing in the domain, which is an asset in the presence of sparse or non-existent rating data (as is the case with one-and-only items, like the trade exhibitions discussed in Section 5).
(2) The integration of the collaborative filtering and content-based paradigms into a single formalism presents an elegant, intuitive and unified synthesis of the problem of recommendation.
(3) The two-sided positive/negative evaluations of the predicted preference of a user for an item allow to take the strength of the arguments into account in the recommendation process.
(4) The proposed framework is generic, and is of interest not only to e-government, but also to e-commerce, e-learning, etc.

For the future, we plan to validate our algorithm on large-scale datasets in an effort to meet the demands of realistic applications like the trade exhibition recommender system. In particular, we will study the impact of particular fuzzy logic operators (t-norms, implicators, aggregation operators) on the recommendation equality, as well as of suitable procedures for ranking the two-valued prediction values according to their fitness for recommendation (as in top-N recommendation). We also plan to incorporate user feedback into the recommender system, in a similar vein as it is done for web search engines (see e.g. [27,28]). In particular, relevance feedback[5] can be very useful to update the positive and negative preference relation, and may help to reduce their associated uncertainty.

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[5] Which may be linguistic, see e.g. [34] for a recent investigation into this matter.

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