On the Representation of Intuitionistic Fuzzy t-Norms and t-Conorms

Glad Deschrijver, Chris Cornelis, and Etienne E. Kerre

***Abstract*—Intuitionistic fuzzy sets form an extension of fuzzy sets: while fuzzy sets give a degree to which an element belongs to a set, intuitionistic fuzzy sets give both a membership degree and a nonmembership degree. The only constraint on those two degrees is that their sum must be smaller than or equal to 1. In fuzzy set theory, an important class of triangular norms and conorms is the class of continuous Archimedean nilpotent triangular norms and conorms. It has been shown that for such t-norms T there exists a permutation φ of [0,1] such that T is the φ-transform of the Łukasiewicz t-norm. In this paper we introduce the notion of intuitionistic fuzzy t-norm and t-conorm, and investigate under which conditions a similar representation theorem can be obtained.**

***Index Terms*—Archimedean property, intuitionistic fuzzy set, intuitionistic fuzzy triangular norm and conorm, nilpotency, representation theorem, φ-transform.**

I. Introduction

FUZZY set theory has been shown to be a useful tool to describe situations in which the data are imprecise or vague. Fuzzy sets handle such situations by attributing a degree to which a certain object belongs to a set. In real life, however, a person may assume that an object x belongs to a set A to a certain degree, but it is possible that he is not so sure about it. In other words, there may be a hesitation or uncertainty about the membership degree of x in A. In fuzzy set theory, there is no means to incorporate that hesitation in the membership degrees. A possible solution is to use intuitionistic fuzzy sets, defined by Atanassov in 1983 [1]. Intuitionistic fuzzy sets give us the possibility to model hesitation and uncertainty by using an additional degree. An intuitionistic fuzzy set A assigns to each element u of the universe U a membership degree $\mu_A(u)(\in [0,1])$ and a nonmembership degree $\nu_A(u)(\in [0,1])$ such that $\mu_A(u) + \nu_A(u) \leq 1$. For all $u \in U$, the value $\pi_A(u) = 1 - \mu_A(u) - \nu_A(u)$ is called the hesitation degree or the intuitionistic index of u to A.

In fuzzy set theory, the nonmembership degree of an element x of the universe is defined as one minus the membership degree (using the standard negation) and thus it is fixed. In intuitionistic fuzzy set theory, the nonmembership degree is more or less independent: the only condition is that it is smaller than one minus the membership degree. Note that both μ_A and ν_A can be seen as fuzzy sets on X (that are not completely independent, because of the condition that the sum of the two degrees should be less than or equal to 1). In this way, the negation of the nonmembership degree w.r.t. the standard fuzzy negation can be seen as a degree of membership. So, for each element $u \in U$ there exist two degrees that model the membership of u in the intuitionistic fuzzy set A, namely $\mu_A(u)$ and $1 - \nu_A(u)$. The length of the interval $[\mu_A(u), 1 - \nu_A(u)]$, which is given by $\pi_A(u)$, can then be seen as a degree modeling the hesitation between the two membership degrees.

An important notion in fuzzy set theory is that of triangular norms and conorms: t-norms and t-conorms are used to define a generalized intersection and union of fuzzy sets, they are applied in the compositional rule of inference (CRI) to obtain the result of the generalized modus ponens (GMP) (see [2] and [3] for an intuitionistic fuzzy set-based approach of the CRI), they are used to define fuzzy inclusion measures [4]–[6]. Triangular norms and conorms serve as aggregation operators, which can be used, e.g., for querying databases (see [7] for an approach via intuitionistic fuzzy sets), to handle multiple rules in the GMP [8], to compute the resulting degree of confidence in a hypothesis when the separate degrees to which the experts support the hypothesis are given [9],... Using intuitionistic fuzzy t-norms the composition of fuzzy relations was extended to the intuitionistic fuzzy case (see [10]–[12]); these compositions are useful in approximate reasoning, e.g., for medical diagnosis and information retrieval (see, e.g., [13] and [14]).

In fuzzy set theory continuous, Archimedean, nilpotent t-norms play a very important role (see, e.g., [15]); they occur for instance in the theory of Łukasiewicz implicators, i.e., fuzzy implicators that fulfill the entire axiom set of Smets and Magrez [16]. A representation theorem was established for continuous Archimedean nilpotent t-norms: a t-norm T is continuous, Archimedean and nilpotent if and only if there exists a permutation φ of [0,1] such that T is the φ-transform of the Łukasiewicz t-norm T_W, i.e. $T = \varphi^{-1} \circ T_W \circ (\varphi \times \varphi)$, where T_W is defined as $T_W(x,y) = \max(0, x+y-1)$, for all $x, y \in [0,1]$, and where $\times$ denotes the product operation [17]. An analogous result holds for t-conorms. In this paper, we extend the notions of t-norm and t-conorm to the intuitionistic fuzzy case, and we generalize said representation theorems to these intuitionistic fuzzy connectives.

II. Intuitionistic Fuzzy Sets

Intuitionistic fuzzy sets were introduced by Atanassov in 1983 and are defined as follows.

Manuscript received October 25, 2002; revised May 9, 2003. The work of C. Cornelis was supported by the Fund for Scientific Research-Flanders.

The authors are with the Department of Mathematics and Computer Science, Fuzziness and Uncertainty Modeling Research Unit, Ghent University, B-9000 Ghent, Belgium (e-mail: glad.deschrijver@UGent.be; chris.cornelis@UGent.ac.be; etienne.kerre@UGent.ac.be).

Digital Object Identifier 10.1109/TFUZZ.2003.822678

1063-6706/04$20.00 © 2004 IEEE

Because of copyright this paper is not presented in its full version here. If you would like to obtain a copy, please e-mail to Chris.Cornelis@UGent.be