Inclusion–Based Approximate Reasoning

Chris Cornelis[1] and Etienne E. Kerre[1]

Department of Mathematics and Computer Science, Ghent University
Fuzziness and Uncertainty Modelling Research Unit
Krijgslaan 281 (S9), B-9000 Gent, Belgium
{chris.cornelis,etienne.kerre}@rug.ac.be
WWW homepage: http://allserv.rug.ac.be/~ekerre

Abstract. We present a kind of fuzzy reasoning, dependent on a measure of *fulfilment* of the antecedent clause, that captures all the expressiveness of the traditional approximate reasoning methodology based on the Compositional Rule of Inference (CRI) and at the same time rules out a good deal of its inherent complexity. We also argue why this approach offers a more genuine solution to the implementation of analogical reasoning than the classically proposed similarity measures.

1 Introduction and Preliminaries

Reasoning with imprecise information expressed as fuzzy sets has received an enormous amount of attention over the last 30 years. More specifically, researchers have undertaken various attempts to model the following reasoning scheme (an extension of the modus ponens logical deduction rule), known as Generalized Modus Ponens (GMP):

$$\begin{array}{ll} \textbf{IF } X \text{ is } A \textbf{ THEN } Y \text{ is } B & (1) \\ \quad X \text{ is } A' & (2) \\ \hline \qquad\qquad\qquad\quad Y \text{ is } B' & (3) \end{array}$$

where X and Y are assumed to be variables taking values in the respective universes U and V; furthermore $A, A' \in \mathcal{F}(U)$ and $B, B' \in \mathcal{F}(V)$[1].

Zadeh suggested to model the if–then rule (1) as a fuzzy relation R (a fuzzy set on $U \times V$) and to apply the Compositional Rule of Inference (CRI) to yield an inference about Y. The CRI is the following inference pattern:

$$\begin{array}{l} X \text{ and } Y \text{ are } R \\ X \text{ is } A' \\ \hline Y \text{ is } R \circ_T A' \end{array}$$

where $\circ_T$ represents the fuzzy composition of R and A' by the t–norm[2] T, i. e. for every $v \in V$ we have:

[1] By $\mathcal{F}(U)$ we denote all fuzzy sets in a universe U.

[2] A t–norm is any symmetric, associative, increasing $[0,1] \times [0,1] \to [0,1]$ mapping T satisfying $T(1, x) = x$ for every $x \in [0,1]$

Because of copyright this paper is not presented in its full version here. If you would like to obtain a copy, please e-mail to Chris.Cornelis@UGent.be