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A BILATTICE-BASED FRAMEWORK FOR HANDLING GRADED TRUTH AND IMPRECISION

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We present a family of algebraic structures, called *rectangular bilattices*, which serve as a natural accommodation and powerful generalization to both intuitionistic fuzzy sets (IFSs) and interval-valued fuzzy sets (IVFSs). These structures are useful on one hand to clarify the exact nature of the relationship between the above two common extensions of fuzzy sets, and on the other hand provide an intuitively attractive framework for the representation of uncertain and potentially conflicting information. We also provide these structures with adequately defined graded versions of the basic logical connectives, and study their properties and relationships. Application potential and intuitive appeal of the proposed framework are illustrated in the context of preference modeling.

Keywords: Bilattices; intuitionistic fuzzy sets; interval-valued fuzzy sets; graded logical connectives; preference modeling.

1. Introduction

As is well-known, fuzzy logic is aimed at handling the concept of partial truth between ‘completely true’ and ‘completely false’ by drawing truth degrees from the unit interval [0,1] (or, more generally, from a complete lattice $\mathcal{L}$, as in Goguen's $\mathcal{L}$-fuzzy logic[1]). Its importance derives from the fact that most modes of human reasoning and especially commonsense reasoning are approximate in nature rather than exact.

Fuzziness, however, cannot adequately cover all the imperfections inherent to real-life situations, since the 'one-dimensional' ordering of truth degrees cannot cope with information deficiency (i.e., a lack or excess of information). In previous papers,[2–4] we have already argued that bilattices (due to Ginsberg[5] and then Fitting[6–9]) can be used to complement the common notion of graded truth (or membership) from ($\mathcal{L}$-)fuzzy set theory, with a notion of graded knowledge. In this sense, bilattices offer a naturally attractive candidate framework for representing graded, uncertain and potentially conflicting information, a claim that is supported also in Refs. 10, 11, 12.

In this paper, we vindicate this thesis by showing that so-called *rectangular bilattices* provide an elegant framework for bridging between intuitionistic fuzzy sets (IFSs)[13] and interval-valued fuzzy sets (IVFSs),[14] which are two frequently encountered and syntactically equivalent generalizations of Zadeh's fuzzy sets. In particular, we will show that Atanassov's decision to restrict the evaluation set for $\mathcal{L}$-intuitionistic fuzzy sets to consistent couples of the "square" $\mathcal{L}^2$ forces the resulting structure to coincide with the "triangle" $\mathcal{I}(\mathcal{L})$ (see Section 2.3).

In the sequel, the developed framework is equipped with suitable implementations for the common logical connectives of negation, conjunction, disjunction and implication. We also demonstrate the robustness of our framework in the context of preference modeling, and show that (rectangular) bilattices are useful not only for encoding the problem statement, but also for its generic solution strategy.

The remainder of this paper is organized as follows: in Section 2 we recall some basic definitions regarding the underlying structures of our framework, namely: bilattices, rectangular bilattices, and derived from them squares and triangles. Then, in Section 3, we show the relationships of these structures to intuitionistic fuzzy sets and interval-valued fuzzy sets. In Section 4 we consider graded versions of the basic logical connectives for reasoning with uncertainty in our setting, and in Section 5 we discuss the application potential of our framework in the context of modeling imprecise preference information. Finally, in Section 6 we conclude.

2. Preliminaries

2.1. *Bilattices*

Definition 1. A *bilattice*[5] is a triple $\mathcal{B} = (B, \leq_t, \leq_k)$, where B is a nonempty set containing at least two elements, and $(B, \leq_t)$, $(B, \leq_k)$ are complete lattices.[1]

[1]Structures that meet this definition are sometimes called *pre-bilattices*. In such cases the notion 'bilattices' is reserved for some particular type of pre-bilattices which is determined according to the way the two partial orders are related; see Definition 2.

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