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REPRESENTABILITY IN INTERVAL-VALUED FUZZY SET THEORY

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Interval-valued fuzzy set theory is an increasingly popular extension of fuzzy set theory where traditional $[0,1]$-valued membership degrees are replaced by intervals in $[0,1]$ that approximate the (unknown) membership degrees. To construct suitable graded logical connectives in this extended setting, it is both natural and appropriate to "reuse" ingredients from classical fuzzy set theory. In this paper, we compare different ways of representing operations on interval-valued fuzzy sets by corresponding operations on fuzzy sets, study their intuitive semantics, and relate them to an existing, purely order-theoretical approach. Our approach reveals, amongst others, that subtle differences in the representation method can have a major impact on the properties satisfied by the generated operations, and that contrary to popular perception, interval-valued fuzzy set theory hardly corresponds to a mere twofold application of fuzzy set theory. In this way, by making the mathematical machinery behind the interval-valued fuzzy set model fully transparent, we aim to foster new avenues for its exploitation by offering application developers a much more powerful and elaborate mathematical toolbox than existed before.

Keywords: Interval-valued fuzzy sets; $\mathcal{L}$-fuzzy sets; graded logical connectives; representability.

1. Introduction

In 1965, Zadeh published his seminal work[1] embodying the commonsense observation that real "objects" can possess a given property to a certain degree; therefore, the property is described by a $[0,1]$-valued membership function attributing to all objects a degree of membership in a fuzzy set. Since that time, the formalism has opened up a vast array of opportunities to intelligent control applications, flexible decision aid systems, approximate reasoning engines, ... As a tool for modelling vagueness, fuzzy logic has become synonymous with high-quality, low-cost solutions to problems whose inherent complexity defies the use of exact, "crisp"