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Advances and challenges in interval-valued fuzzy logic

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Abstract

Among the various extensions to the common [0, 1]-valued truth degrees of "traditional" fuzzy set theory, *closed intervals of* [0, 1] stand out as a particularly appealing and promising choice for representing imperfect information, nicely accommodating and combining the facets of vagueness and uncertainty without paying too much in terms of computational complexity. From a logical point of view, due to the failure of the omnipresent prelinearity condition, the underlying algebraic structure $\mathscr{L}^I$ falls outside the mainstream of the research on formal fuzzy logics (including MV-, BL- and MTL-algebras), and consequently so far has received only marginal attention. This comparative lack of interest for interval-valued fuzzy logic has been further strengthened, perhaps, by taking for granted that its algebraic operations amount to a twofold application of corresponding operations on the unit interval. Abandoning that simplifying assumption, however, we may find that $\mathscr{L}^I$ reveals itself as a very rich and noteworthy structure allowing the construction of complex and surprisingly well-behaved logical systems. Reviewing the main advances on the algebraic characterization of logical operations on $\mathscr{L}^I$, and relating these results to the familiar completeness questions (which remain as major challenges) for the associated formal fuzzy logics, this paper paves the way for a systematic study of interval-valued fuzzy logic in the narrow sense.

Keywords: Interval-valued fuzzy logic; Logical connectives; Algebraic structures; Representability; Fuzzy logics

1. Introduction

Interval-valued fuzzy set theory (apparently introduced independently in the mid-seventies by Grattan-Guinness [11], Jahn [14], Sambuc [19] and Zadeh [21]) is an increasingly popular extension of fuzzy set theory where traditional [0, 1]-valued membership degrees are replaced by intervals in [0, 1] that approximate the (partially unknown) exact degrees. Hence, not only vagueness (lack of sharp class boundaries), but also a feature of uncertainty (lack of information) can be addressed intuitively. Moreover, interval-valued fuzzy sets (IVFSs) are considerably easier to handle in practice than the similarly inspired type-2 fuzzy sets (of which IVFSs are in fact a special case, called "interval type-2 fuzzy sets" in that context, see e.g. [17]); as further evidence of their wide relevance, they also subsume the syntactically equivalent frameworks of Atanassov's intuitionistic fuzzy sets [1] and of Gau and Buehrer's vague sets [7].

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